

C1 ALGEBRA

Answers - Worksheet H

1 a $= 2x(10 - x - 3x^2)$
 $= 2x(2 + x)(5 - 3x)$
 b $2x(2 + x)(5 - 3x) = 0$
 $x = -2, 0$ or $\frac{5}{3}$

3 a $x^2 - 5 = 4x$
 $x^2 - 4x - 5 = 0$
 $(x + 1)(x - 5) = 0$
 $x = -1$ or 5
 b $9 - (5 - x) = 2x(5 - x)$
 $2x^2 - 9x + 4 = 0$
 $(2x - 1)(x - 4) = 0$
 $x = \frac{1}{2}$ or 4

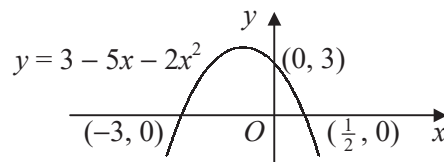
5 $x = \frac{-5\sqrt{2} \pm \sqrt{50 + 48}}{4}$
 $= \frac{-5\sqrt{2} \pm \sqrt{98}}{4}$
 $= \frac{-5\sqrt{2} \pm 7\sqrt{2}}{4}$
 $= -3\sqrt{2}$ or $\frac{1}{2}\sqrt{2}$

7 $y^2 - 10y + 16 = 0$
 $(y - 2)(y - 8) = 0$
 $y = 2^x = 2$ or 8
 $x = 1$ or 3

9 a $f(x) = -[x^2 - 4x] + 3$
 $= -[(x - 2)^2 - 4] + 3$
 $= -(x - 2)^2 + 7$
 b turning point is $(2, 7)$
 c $-(x - 2)^2 + 7 = 2$
 $(x - 2)^2 = 5$
 $x = 2 \pm \sqrt{5}$

2 a $AB^2 = (6 + 2)^2 + (k - 1)^2 = 64 + k^2 - 2k + 1$
 $= k^2 - 2k + 65$
 b $k^2 - 2k + 65 = 10^2 = 100$
 $k^2 - 2k - 35 = 0$
 $(k + 5)(k - 7) = 0$
 $k = -5$ or 7

4 a $y = -2[x^2 + \frac{5}{2}x] + 3$
 $= -2[(x + \frac{5}{4})^2 - \frac{25}{16}] + 3$
 $= -2(x + \frac{5}{4})^2 + \frac{49}{8}$
 \therefore turning point is $(-\frac{5}{4}, \frac{49}{8})$
 b $3 - 5x - 2x^2 = 0$
 $2x^2 + 5x - 3 = 0$
 $(2x - 1)(x + 3) = 0, x = -3$ or $\frac{1}{2}$



6 a $y = 3[x^2 - 3x] + k = 3[(x - \frac{3}{2})^2 - \frac{9}{4}] + k$
 $= 3(x - \frac{3}{2})^2 - \frac{27}{4} + k$
 \therefore x -coordinate of $P = \frac{3}{2}$
 b y -coord of $P = k - \frac{27}{4} = \frac{17}{4} \therefore k = 11$
 \therefore curve is $y = 3x^2 - 9x + 11$
 \therefore coordinates of Q are $(0, 11)$

8 equal roots $\therefore b^2 - 4ac = 0$
 $4 - 4k(3 - 2k) = 0$
 $2k^2 - 3k + 1 = 0$
 $(2k - 1)(k - 1) = 0$
 $k = \frac{1}{2}$ or 1

10 a $x = \frac{5 \pm \sqrt{25 - 12}}{6}$
 $= \frac{1}{6}(5 \pm \sqrt{13})$
 b $x(x - 1) = 3(x + 2)$
 $x^2 - 4x - 6 = 0$
 $x = \frac{4 \pm \sqrt{16 + 24}}{2} = \frac{4 \pm 2\sqrt{10}}{2}$
 $= 2 \pm \sqrt{10}$

11 a $(x - 2k)^2 - 4k^2 + 6 = 0$

$$(x - 2k)^2 = 4k^2 - 6$$

$$x - 2k = \pm\sqrt{4k^2 - 6}$$

$$x = 2k \pm \sqrt{4k^2 - 6}$$

b $k = 3$

$$\therefore x = 6 \pm \sqrt{36 - 6}$$

$$= 6 \pm \sqrt{30}$$

12 a $x^2 - 6x - 3 = 0$

$$x = \frac{6 \pm \sqrt{36 + 12}}{2} = \frac{6 \pm 4\sqrt{3}}{2}$$

$$= 3 \pm 2\sqrt{3}$$

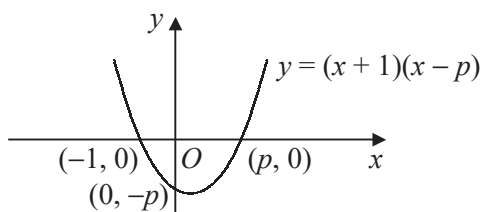
b $y(2y^2 + y - 15) = 0$

$$y(2y - 5)(y + 3) = 0$$

$$y = -3, 0 \text{ or } \frac{5}{2}$$

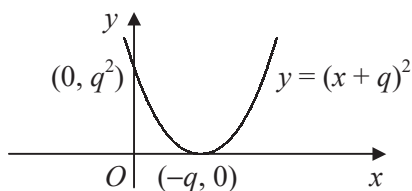
13 a $x = 0 \Rightarrow y = -p$

$$y = 0 \Rightarrow x = -1 \text{ or } p$$



b $x = 0 \Rightarrow y = q^2$

$$y = 0 \Rightarrow x = -q \quad [-q > 0]$$



15 a $x^{\frac{2}{3}} = (x^{\frac{1}{3}})^2 = t^2$

b let $t = x^{\frac{1}{3}} \Rightarrow 2t^2 + t - 6 = 0$
 $(2t - 3)(t + 2) = 0$
 $t = -2 \text{ or } \frac{3}{2}$

but $x = t^3 \therefore x = -8 \text{ or } \frac{27}{8}$

16 a $= (k - 4)^2 - 16 + 20$

$$= (k - 4)^2 + 4$$

b $x^2 - kx + 2k - 5 = 0$

$$\text{discriminant} = b^2 - 4ac$$

$$= k^2 - 4(2k - 5)$$

$$= k^2 - 8k + 20$$

using a $= (k - 4)^2 + 4$

for all real k , $(k - 4)^2 \geq 0$

$$\therefore \text{discriminant} > 0$$

\therefore real and distinct roots for all real k

17 a $(x^2 + 2x - 3)(x^2 - 3x - 4) \equiv x^2(x^2 - 3x - 4) + 2x(x^2 - 3x - 4) - 3(x^2 - 3x - 4)$

$$\equiv x^4 - 3x^3 - 4x^2 + 2x^3 - 6x^2 - 8x - 3x^2 + 9x + 12$$

$$\equiv x^4 - x^3 - 13x^2 + x + 12$$

b $(x^2 + 2x - 3)(x^2 - 3x - 4) = 0$

$$(x + 3)(x - 1)(x + 1)(x - 4) = 0$$

$$x = -3, -1, 1 \text{ or } 4$$