Arithmetic Series - Past Edexcel Exam Questions

1. The rth term of an arithmetic series is $(2r - 5)$.

   (a) Write down the first 3 terms of the series. [2]

   (b) State the value of the common difference. [1]

   (c) Show that $\sum_{r=1}^{n}(2r - 5) = n(n - 4)$. [3]

Question 5 - Jan 2005

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2. An arithmetic series has first term a and common difference d.

   (a) Prove that the sum of the first n terms of the series is

   $$\frac{1}{2}n\left[2a + (n-1)d\right]$$

   [4]

   Sean repays a loan over a period of n months. His monthly repayments form an arithmetic sequence.

   He repays £149 in the first month, £147 in the second month, £145 in the third month, and so on. He makes his final payment in the nth month, where $n > 21$.

   (b) Find the amount Sean repays in the 21st month. [2]

   Over the n months, he repays a total of £5000.

   (c) Form an equation in n, and show that your equation may be written as

   $$n^2 - 150n + 5000 = 0$$

   . [3]

   (d) Solve the equation in part (c). [3]

   (e) State, with reason, which of the solutions to the equation in part (c) is **not** a sensible solution to the repayment problem. [1]

Question 9 - May 2005

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3. On Alice's 11th birthday she started to receive an annual allowance. The first annual allowance was £500 and on each following birthday the allowance was increased by £200.

   (a) Show that, immediately after her 12th birthday, the total of the allowances that Alice had received was £1200. [1]

   (b) Find the amount of Alice's annual allowance on her 18th birthday. [2]

   (c) Find the total of the allowances that Alice had received up to and including her 18th birthday. [3]

   When the total of the allowances that Alice had received reached £32,000 the allowance stopped.

   (d) Find how old Alice was when she received her last allowance. [7]

Question 7 - Jan 2006

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4. An athlete prepares for a race by completing a practice run on each of 11 consecutive days. On each day after the first day he runs further than he ran on the previous day. The lengths of his 11 practice runs form an arithmetic sequence with first term a km and common difference d km.

   He runs 9km on the 11th day, and he runs a total of 77km over the 11 day period.

   Find the value of a and the value of d. [7]

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Question 7 - May 2006

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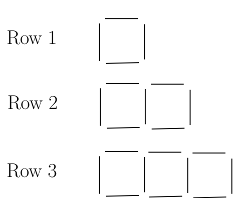
5. Ann has some sticks that are all of the same length. She arranges them in squares and has made the following 3 rows of patterns:



   She notices that 4 sticks are required to make the single square in the first row, 7 sticks to make the 2 squares in the second row and in the third row she needs 10 sticks to make 3 squares.

(a) Find an expression, in terms of n, for the number of sticks required to make a similar arrangement of n squares in the nth row. [3]

Ann continues to make squares following the same pattern. She makes 4 squares in the 4th row and so on until she has completed 10 rows.

(b) Find the total number of sticks Ann uses in making these 10 rows. [3]

Ann started with 1750 sticks. Given that Ann continues the pattern to complete k rows but does not have sufficient sticks to complete the $(k+1)$th row,

(c) show that k satisfies $(3k-100)(k+35) < 0$. [4]

(d) Find the value of k. [2]

Question 9 - Jan 2007

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6. A girl saves money over a period of 200 weeks. She saves 5p in Week 1, 7p in Week 2, 9p in Week 3, and so on until Week 200. Her weekly savings form an arithmetic sequence.

(a) Find the amount she saves in Week 200. [3]

(b) Calculate her total savings over the complete 200 week period. [3]

Question 4 - May 2007

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7. The first term of an arithmetic sequence is 30 and the common difference is -1.5.

(a) Find the value of the 25th term. [2]

The rth term of the sequence is 0.

(b) Find the value of r. [2]

The sum of the first n terms of the sequence is S_n.

(c) Find the largest positive value of S_n. [3]

Question 11 - Jan 2008

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8. Sue is training for a marathon. Her training includes a run every Saturday starting with a run of 5km on the first Saturday. Each Saturday she increases the length of her run from the previous Saturday by 2km.

   (a) Show that on the 4th Saturday of training she runs 11km. [1]

   (b) Find an expression, in term of n, for the length of her training run on the nth Saturday. [2]

   (c) Show that the total distance she runs on Saturday in n weeks of training is $n(n+4)$km. [3]

   On the nth Saturday Sue runs 43km.

   (d) Find the value of n. [2]

   (e) Find the total distance, in km, Sue runs on Saturdays in n weeks of training. [2]

Question 7 - June 2008

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9. The first term of an arithmetic series is a and the common difference is d.
   The 18th term of the series is 25 and the 21st term of the series is $32\frac{1}{2}$.

   (a) Use this information to write down two equations in a and d. [2]

   (b) Show that $a = -17.5$ and find the value of d. [2]

   The sum of the first n terms of the series is 2750.

   (c) Show that n is given by

   $$n^2 - 15n = 55 \times 40.$$

   [4]

   (d) Hence find the value of n. [3]

Question 9 - Jan 2009

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10. A 40-year building programme for new houses began in Oldtown in the year 1951 (Year 1) and finished in 1990 (Year 40).

    The numbers of houses built each year form an arithmetic sequence with first term a and common difference d.

    Given that 2400 new houses were built in 1960 and 600 new houses were built in 1990, find

(a) the value of d, [3]

(b) the value of a, [2]

(c) the total number of houses built in Oldtown over the 40-year period. [3]

Question 5 - June 2009

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11. Jill gave money to a charity over a 20-year period, from Year 1 to Year 20 inclusive. She gave £150 in Year 1, £160 in Year 2, £170 in Year 3, and so on, so that the amounts of money she gave each year formed an arithmetic sequence.

(a) Find the amount of money she gave in Year 10, [2]

(b) Calculate the total amount of money she gave over the 20-year period. [3]

Kevin also gave money to charity over the same 20-year period.

He gave £A in Year 1 and the amounts of money he gave each year increased, forming an arithmetic sequence with common difference £30.

The total amount of money that Kevin gave over the 20-year period was **twice** the total amount of money that Jill gave.

(c) Calculate the value of A. [4]

Question 7 - Jan 2010

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12. A farmer has a pay scheme to keep fruit pickers working throughout the 30 day season. He pays £a on their first day, £$(a + d)$ for their second day, £$(a + 2d)$ on their third day, and so on, thus increasing the daily payment by £d for each extra day they work.

A picker who works for all 30 days will earn £40.75 on the final day.

(a) Use this information to form an equation in a and d. [2]

A picker who works for all 30 days will earn a total of £1005.

(b) Show that $15(a + 40.75) = 1005$. [2]

(c) Hence find the value of a and the value of d. [4]

Question 9 - May 2010

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13. An arithmetic sequence has first term a and common difference d. The sum of the first 10 terms of the sequence is 162.

(a) Show that $10a + 45d = 162$. [2]

Given also that the sixth term of the sequence is 17,

(b) write down a second equation in a and d, [1]

(c) find the value of a and the value of d. [4]

Question 6 - Jan 2011

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14. (a) Calculate the sum of all the even numbers from 2 to 100 inclusive,

$$2 + 4 + 6 + ... + 100$$

[3]

(b) In the arithmetic series

$$k + 2k + 3k + ... + 100,$$

where k is a positive integer and k is a factor of 100.

i. Find, in terms of k, an expression for the number of terms in this series.

ii. Show that the sum of this series is

$$50 + \frac{5000}{k}$$

[4]

(c) Find, in terms of k, the 50th term of the arithmetic sequence

$$(2k + 1), (4k + 4), (6k + 7), ...,$$

giving your answer in its simplest form. [4]

Question 9 - May 2011

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15. A company offers two salary schemes for a 10-year period, Year 1 to Year 10 inclusive.

Scheme 1: Salary in Year 1 is £P. Salary increases by £$(2T)$ each year, forming an arithmetic sequence.

Scheme 2: Salary is £$(P + 1800)$. Salary increases by £T each year, forming an arithmetic sequence.

(a) Show that the total earned under Salary Scheme 1 for the 10-year period is

$$£(10P + 90T)$$

[2]

For the 10-year period, the total earned is the same for both salary schemes.

(b) Find the value of T. [4]

For this value of T, the salary in Year 10 under Salary Scheme 2 is £29,850.

(c) Find the value of P. [3]

Question 9 - Jan 2012

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16. A boy saves some money over a period of 60 weeks. He saves 10p in week 1, 15p in week 2, 20p in week 3 and so on until week 60. His weekly savings form an arithmetic sequence.

(a) Find how much he saves in week 15. [2]

(b) Calculate the total amount he saves over the 60 week period. [3]

The boy's sister also saves some money each week over a period of m weeks. She saves 10p in week 1, 20p in week 2, 30p in week 3 and so on that her weekly savings form an arithmetic sequence. She saves a total of £63 in the m weeks.

(c) Show that

$$m(m + 1) = 35 \times 36.$$

[4]

(d) Hence write down the value of m. [1]

Question 6 - May 2012

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17. Lewis played a game of space invaders. He scored points for each spaceship that he captured.

Lewis scored 140 points for capturing his first spaceship.

He scored 160 points for capturing his second spaceship, 180 points for capturing his third spaceship, and so on.

The number of points scored for capturing each successive formed an arithmetic sequence.

(a) Find the number of points that Lewis scored for capturing his 20th spaceship.
. [2]

(b) Find the total number of points Lewis scored for capturing his first 20 spaceships.
. [3]

Sian played an adventure game. She scored points for each dragon that she captured. The number of points that Sian scored for capturing each successive dragon formed an arithmetic sequence.

Sian captured n dragons and the total number of points that she scored for capturing all n dragons was 8500.

Given that Sian scored 300 points for capturing her first dragon and then 700 points for capturing her nth dragon,

(c) find the value of n. [3]

Question 7 - Jan 2013

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18. A company, which is making 200 mobile phones each week, plans to increase its production.

The number of mobile phones produced is to be increased by 20 each week from 200 in week 1 to 220 in week 2, to 240 in week 3 and so on, until it is producing 600 in week N.

(a) Find the value of N. [2]

The company then plans to make 600 mobiles each week.

(b) Find the total number of mobile phones that will be made in the first 52 weeks starting from and including week 1. [5]

Question 7 - Jun 2013

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19. In the year 2000 a shop sold 150 computers. Each year the shop sold 10 more computers than the year before, so that the shop sold 160 computers in 2001, 170 computers in 2002, and so on forming an arithmetic sequence.

(a) Show that the shop sold 220 computers in 2007. [2]

(b) Calculate the total number of computers the shop sold from 2000 to 2013 inclusive.
. [3]

In the year 2000, the selling price of each computer was £900. The selling price fell by £20 each year, so that in 2001 the selling price was £880, in 2002 the selling price was £860, and so on forming an arithmetic sequence.

(c) In a particular year, the selling price of each computer in £s was equal to three times the number of computers the shop sold in that year. By forming and solving an equation, find the year in which this occurred. [4]

Question 8 - Jun 2014

Solutions

1. (a) -3,-1,1
   (b) 2
   (c) -
2. (a) -
   (b) $u_{21} = £109$
   (c) -
   (d) $n = 50$, $n = 100$
   (e) $n \neq 100$ since $u_{50} > 0$ and $u_{100} < 0$
3. (a) -
   (b) $U_8 = 1900$
   (c) $S_8 = 9600$
   (d) 26 years old
4. $d = \frac{2}{5}$, $a = 5$
5. (a) $U_n = 3n + 1$
   (b) $S_{10} = 175$
   (c) -
   (d) $k = 33$
6. (a) $U_{200} = £4.03$
   (b) $S_{200} = £408.00$
7. (a) $U_{25} = -6$
   (b) $r = 21$
   (c) $S_{21} = 315$
8. (a) -
   (b) $U_n = 3 + 2n$
   (c) -
   (d) $n = 20$
   (e) $S_{20} = 480$km

9. (a) $a + 17d = 25$, $a + 10d = 32.5$
   (b) $d = 2.5$
   (c) -
   (d) $n = 55$
10. (a) $d = -60$
    (b) $a = 2940$
    (c) $S_{40} = 70,800$
11. (a) $U_{10} = £240$
    (b) $S_{20} = £4900$
    (c) $A = 205$
12. (a) $a + 29d = 40.75$
    (b) -
    (c) $a = 26.25$, $d = 0.5$
13. (a) -
    (b) $a + 5d = 17$
    (c) $d = \frac{8}{5}$, $a = 9$
14. (a) $S_{50} = 2550$
    (b) i. $\frac{100}{k}$
        ii. -
    (c) $U_{50} = 100k + 148$
15. (a) -
    (b) $T = 400$
    (c) $P = £24,450$
16. (a) $U_{15} = 80p$
    (b) $S_{60} = £94.50$
    (c) -
    (d) $m = 35$
17. (a) $U_{20} = 520$

(b) $S_{20} = 6600$

(c) $n = 17$

18. (a) $N = 21$

(b) $S_{21} = 27,000$

19. (a) -

(b) $S_{14} = 3010$

(c) 2009

