Differentiation - Past Edexcel Exam Questions

1. (a) Given that $y = 5x^3 + 7x + 3$, find

   i. $\frac{dy}{dx}$ [3]

   ii. $\frac{d^2y}{dx^2}$ [1]

   .

Question 2ai, 2aii - January 2005

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2. The curve C has equation $y = 4x^2 + \frac{5-x}{x}$, $x \neq 0$. The point P on C has x-coordinate 1.

   (a) Show that the value of $\frac{dy}{dx}$ at P is 3. [5]

   (b) Find an equation of the tangent to C at P. [3]

   This tangent meets the x-axis at the point $(k, 0)$.

   (c) Find the value of k. [2]

Question 7 - January 2005

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3. Given that $y = 6x - \frac{4}{x^2}$, $x \neq 0$,

   (a) find $\frac{dy}{dx}$, [2]

Question 2a - May 2005

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4. The curve C has equation $y = \frac{1}{3}x^3 - 4x^2 + 8x + 3$.

   The point P has coordinates $(3, 0)$.

   (a) Show that P lies on C. [1]

   (b) Find the equation of the tangent to C at P, giving your answer in the form $y = mx + c$, where m and c are constants. [5]

   Another point Q also lies on C. The tangent to C at Q is parallel to the tangent to C at P.

(c) Find the coordinates of Q. [5]

Question 10 - May 2005

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5. Given that $y = 2x^2 - \frac{6}{x^3}$, $x \neq 0$,

(a) find $\frac{dy}{dx}$, [2]

Question 4 - January 2006

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6. .

Figure 1:

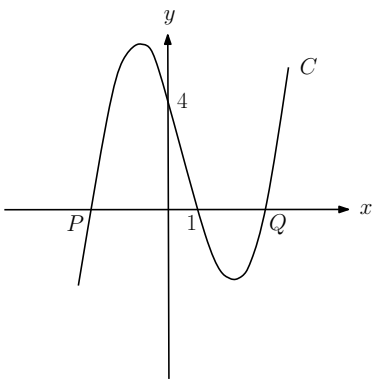

Figure 1 shows part of the curve C with equation

$$y = (x - 1)\left(x^2 - 4\right).$$

The curve cuts the x-axis at the points P, (1,0) and Q as shown in Figure 1.

(a) Write down the x-coordinate of P and the x-coordinate of Q. [2]

(b) Show that $\frac{dy}{dx} = 3x^2 - 2x - 4$. [3]

(c) Show that $y = x + 7$ is an equation of the tangent to C at the point $(-1, 6)$. [2]

The tangent to C at the point R is parallel to the tangent at the point $(-1, 6)$.

(d) Find the exact coordinates of R. [5]

Question 9 - January 2006

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7. Differentiate with respect to x

   (a) $x^4 + 6\sqrt{x}$, [3]

   (b) $\frac{(x+4)^2}{x}$. [4]

Question 5 - May 2006

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8. Given that

$$y = 4x^3 - 1 + 2x^{\frac{1}{2}}, \quad x > 0,$$

   find $\frac{dy}{dx}$. [4]

   .

Question 1 - January 2007

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9. The curve C has equation $y = 4x + 3x^{\frac{3}{2}} - 2x^2$, $x > 0$.

   (a) Find an expression for $\frac{dy}{dx}$. [3]

   (b) Show that the point $P(4, 8)$ lies on C. [1]

   (c) Show that an equation of the normal to C at the point P is

$$3y = x + 20.$$

   [4]

   The normal to C at P cuts the x-axis at the point Q.

   (d) Find the length PQ, giving your answer in a simplified surd form. [3]

Question 8 - January 2007

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10. Given that $y = 3x^2 + 4\sqrt{x}$, $x > 0$, find

    (a) $\frac{dy}{dx}$, [2]

    (b) $\frac{d^2y}{dx^2}$. [2]

Question 3a,b - May 2007

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11. The curve C has equation $y = x^2(x-6) + \frac{4}{x}$, $x > 0$.

The points P and Q lie on C and have x-coordinates 1 and 2 respectively.

(a) Show that the length of PQ is $\sqrt{170}$. [4]

(b) Show that the tangents to C at P and Q are parallel. [5]

(c) Find an equation for the normal to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [4]

Question 10 - May 2007

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12. (a) Write $\frac{2\sqrt{x}+3}{x}$ in the form $2x^p + 3x^q$, where p and q are constants. [2]

Given that $y = 5x - 7 + \frac{2\sqrt{x}+3}{x}$, $x > 0$,

(b) find $\frac{dy}{dx}$, simplifying the coefficient of each term. [4]

Question 5 - January 2008

13.

$$f(x) = 3x + x^3, \quad x > 0.$$

(a) Differentiate to find $f'(x)$. [2]

Given that $f'(x) = 15$,

(b) find the value of x. [3]

Question 4 - June 2008

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14. The curve C has equation $y = kx^3 - x^2 + x - 5$, where k is a constant.

(a) Find $\frac{dy}{dx}$. [2]

The point A with x-coordinate $-\frac{1}{2}$ lies on C. The tangent to C at A is parallel to the line with equation $2y - 7x + 1 = 0$.

Find

(b) the value of k, [4]

(c) the value of the y-coordinate of A. [2]

Question 9 - June 2008

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15. Given that $\frac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$ can be written in the form $2x^p - x^q$,

(a) write down the value of p and the value of q. [2]

Given that $y = 5x^4 - 3 + \frac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$, $x > 0$,

(b) find $\frac{dy}{dx}$, simplifying the coefficient of each term. [4]

Question 6 - January 2009

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16. The curve C has equation

$$y = 9 - 4x - \frac{8}{x}, \quad x > 0.$$

The point P on C has x-coordinate equal to 2.

(a) Show that the equation of the tangent to C at the point P is $y = 1 - 2x$. [6]

(b) Find an equation of the normal to C at the point P. [3]

The tangent at P meets the x-axis at A and the normal at P meets the x-axis at B.

(c) Find the area of the triangle APB. [4]

Question 11 - January 2009

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17. Given that $y = 2x^3 + \frac{3}{x^2}$, $x \neq 0$, find

(a) $\frac{dy}{dx}$, [3]

Question 3a - June 2009

18.

$$f(x) = \frac{(3-4\sqrt{x})^2}{\sqrt{x}}, \quad x > 0$$

(a) Show that $f(x) = 9x^{-\frac{1}{2}} + Ax^{\frac{1}{2}} + B$, where A and B are constants to be found. [3]

(b) Find $f'(x)$. [3]

(c) Evaluate $f'(9)$. [2]

Question 9 - June 2009

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19. The curve C has equation

$$y = x^3 - 2x^2 - x + 9, \quad x > 0$$

The point P has coordinates (2,7).

(a) Show that P lies on C. [1]

(b) Find the equation of the tangent to C at P, giving your answer in the form $y = mx + c$, where m and c are constants. [5]

The point Q also lies on C.

Given that the tangent to C at Q is perpendicular to the tangent to C at P,

(c) show that the x-coordinate of Q is $\frac{1}{3}(2+\sqrt{6})$. [5]

Question 11 - June 2009

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20. Given that $y = x^4 + x^{\frac{1}{3}} + 3$, find $\frac{dy}{dx}$. [3]

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Question 1 - January 2010

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21. The curve C has equation

$$y = \frac{(x+3)(x-8)}{x}, \quad x > 0.$$

(a) Find $\frac{dy}{dx}$ in its simplest form. [4]

(b) Find an equation of the tangent to C at the point where $x = 2$. [4]

Question 6 - January 2010

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22. Given that

$$y = 8x^3 - 4\sqrt{x} + \frac{3x^2+2}{x}, \quad x > 0,$$

find $\frac{dy}{dx}$. [6]

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Question 7 - May 2010

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23. The curve C has equation

$$y = \frac{1}{2}x^3 - 9x^{\frac{3}{2}} + \frac{8}{x} + 30, \quad x > 0.$$

(a) Find $\frac{dy}{dx}$. [4]

(b) Show that the point $P(4, -8)$ lies on C. [2]

(c) Find an equation of the normal to C at the point P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [6]

Question 11 - January 2011

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24. Given that $y = 2x^5 + 7 + \frac{1}{x^3}$, $x \neq 0$, find, in their simplest form,

(a) $\frac{dy}{dx}$, [3]

Question 2a - May 2011

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25. The curve C has equation

$$y = (x+1)(x+3)^2.$$

(a) Sketch C, showing the coordinates of the points at which C meets the axes. [4]

(b) Show that $\frac{dy}{dx} = 3x^2 + 14x + 15$. [3]

The point A, with x-coordinate -5, lies on C.

(c) Find the equation of the tangent to C at A, giving your answer in the form $y = mx + c$, where m and c are constants. [4]

Another point B also lies on C. The tangents to C at A and B are parallel.

(d) Find the x-coordinate of B. [3]

Question 10 - June 2011

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26. Given that $y = x^4 + 6x^{\frac{1}{2}}$, find in their simplest form

(a) $\frac{dy}{dx}$, [3]

Question 1a - January 2012

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27. The curve C_1 has equation

$$y = x^2(x+2)$$

(a) Find $\frac{dy}{dx}$. [2]

(b) Sketch C_1, showing the coordinates of the points where C_1 meets the x-axis. [3]

The curve C_2 has equation

$$y = (x-k)^2(x-k+2)$$

where k is a constant and $k > 2$.

(c) Find the gradient of C_1 at each point where C_1 meets the x-axis. [2]

(d) Sketch C_2, showing the coordinates of the points where C_2 meets the x and y axes.

. [3]

Question 8 - January 2012

28.

$$y = 5x^3 - 6x^{\frac{4}{3}} + 2x - 3.$$

(a) Find $\frac{dy}{dx}$, giving each term in its simplest form. [4]

(b) Find $\frac{d^2y}{dx^2}$. [2]

Question 4 - May 2012

29. The curve C has equation

$$y = 2x - 8\sqrt{x} + 5, \quad x \geq 0$$

(a) Find $\frac{dy}{dx}$, giving each term in its simplest form. [3]

The point P on C has x-coordinate equal to $\frac{1}{4}$.

(b) Find the equation of the tangent to C at the point P, giving your answer in the form $y = ax + b$, where a and b are constants. [4]

The tangent to C at the point Q is parallel to the line with equation $2x - 3y + 18 = 0$.

(c) Find the coordinates of Q. [5]

Question 11 - January 2013

30.

$$f'(x) = \frac{(3 - x^2)^2}{x^2}, \quad x \neq 0$$

(a) Show that

$$f'(x) = 9x^{-2} + A + Bx^2,$$

where A and B are constants to be found. [3]

(b) Find $f''(x)$. [2]

Question 19a,b - May 2013

31. Differentiate with respect to x, giving each answer in its simplest form.

(a) $(1-2x)^2$ [3]

(b) $\frac{x^5+6\sqrt{x}}{2x^2}$ [4]

Question 7 - May 2014

Solutions

1. (a) i. $15x^2 + 7$
   ii. $30x$

2. (a) -
   (b) $y = 3x + 5$
   (c) $k = -\frac{5}{3}$

3. (a) $6 + 8x^{-3}$

4. (a) -
   (b) $y = -7x + 21$
   (c) $\left(5, -\frac{46}{3}\right)$

5. (a) $4x + 18x^{-4}$

6. (a) -2,2
   (b) -
   (c) -
   (d) $\left(\frac{5}{3}, -\frac{22}{27}\right)$

7. (a) $4x^3 + 3x^{-\frac{1}{2}}$
   (b) $1 - 16x^{-2}$

8. $12x^2 + x^{-\frac{1}{2}}$

9. (a) $4 + \frac{9}{2}x^{\frac{1}{2}} - 4x$
   (b) -
   (c) -
   (d) $8\sqrt{10}$

10. (a) $6x + 2x^{-\frac{1}{2}}$
    (b) $6 - x^{-\frac{3}{2}}$

11. (a) -
    (b) -
    (c) $x - 13y - 14 = 0$

12. (a) $2x^{-\frac{1}{2}} + 3x^{-1}$
    (b) $5 - x^{-\frac{3}{2}} - 3x^{-2}$

13. (a) $3 + 3x^2$
    (b) $x = 2$

14. (a) $3kx^2 - 2x + 1$
    (b) $k = 2$
    (c) -6

15. (a) $p = \frac{3}{2}$, $q = 1$
    (b) $20x^3 + 3x^{\frac{1}{2}} - 1$

16. (a) -
    (b) $y = \frac{1}{2}x - 4$
    (c) $\frac{45}{4}$

17. (a) $6x^2 - 6x^{-3}$

18. (a) $A = 16$, $B = -24$
    (b) $-\frac{9}{2}x^{-\frac{3}{2}} + 8x^{-\frac{1}{2}}$
    (c) $\frac{5}{2}$

19. (a) -
    (b) $y = 3x + 1$
    (c) -

20. $4x^2 + \frac{1}{3}x^{-\frac{2}{3}}$

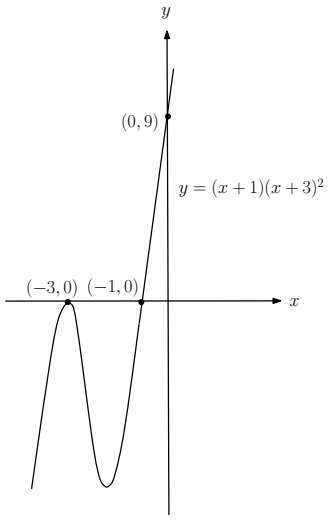
21. (a) $1 + 24x^{-2}$
    (b) $y = 7x - 29$

22. $24x^2 - 2x^{-\frac{1}{2}} + 3 - 2x^{-2}$

23. (a) $\frac{3}{2}x^2 - \frac{27}{2}x^{\frac{1}{2}} - 8x^{-2}$
    (b) -
    (c) $2x - 7y - 64 = 0$

24. $10x^4 - 3x^{-4}$

25. (a) .



(b) -

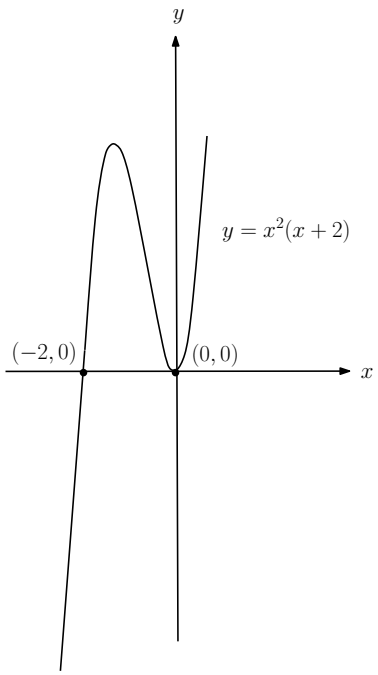
(c) $y = 20x + 84$

(d) $x = \frac{1}{3}$

26. $4x^3 + 3x^{-\frac{1}{2}}$

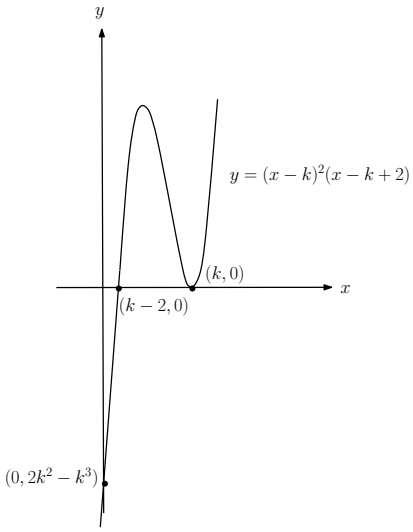
27. (a) $3x^2 + 4x$

(b) .



(c) 4,0

(d) .



28. (a) $15x^2 - 8x^{\frac{1}{3}} + 2$

    (b) $30x - \frac{8}{3}x^{-\frac{2}{3}}$

29. (a) $2 - 4x^{-\frac{1}{2}}$

    (b) $y = -6x + 3$

    (c) $(9, -1)$

30. (a) $A = -6,\ B = 1$

    (b) $-18x^{-3} + 2x$

31. (a) $-4 + 8x$

    (b) $\frac{3}{2}x^2 - \frac{9}{2}x^{-\frac{5}{2}}$