C2 SEQUENCES AND SERIES *Worksheet C*

1 Expand each of the following, simplifying the coefficient in each term.

a $(1+x)^4$ **b** $(1-x)^5$ **c** $(1+4x)^3$ **d** $(1-2y)^3$

e $(1+\frac{1}{2}x)^4$ **f** $(1+\frac{1}{3}y)^3$ **g** $(1+x^2)^5$ **h** $(1-\frac{3}{2}x)^4$

2 Expand each of the following, simplifying the coefficient in each term.

a $(x+y)^3$ **b** $(a-b)^5$ **c** $(x+2y)^4$ **d** $(2+y)^3$

e $(3-x)^3$ **f** $(5+2x)^4$ **g** $(3-4y)^5$ **h** $(3+\frac{1}{2}x)^4$

3 Find the first four terms in the expansion in ascending powers of x of

a $(1+x)^{10}$ **b** $(1-x)^6$ **c** $(1+2x)^8$ **d** $(1-\frac{1}{2}x)^7$

e $(1+x^3)^6$ **f** $(2+x)^9$ **g** $(3-x)^7$ **h** $(2+5x)^{10}$

4 Find the coefficient indicated in the following expansions.

a $(1+x)^{20}$, coefficient of x^3 **b** $(1-x)^{14}$, coefficient of x^4

c $(1+4x)^9$, coefficient of x^2 **d** $(1-3y)^{14}$, coefficient of y^3

e $(1-\frac{1}{3}x)^{12}$, coefficient of x^4 **f** $(1-\frac{1}{2}x)^{16}$, coefficient of x^5

g $(1+\frac{2}{5}x)^{15}$, coefficient of x^2 **h** $(1+y^2)^8$, coefficient of y^6

5 Express each of the following in the required form where a and b are integers.

a $(1+\sqrt{5})^3$ in the form $a+b\sqrt{5}$ **b** $(1-\sqrt{3})^4$ in the form $a+b\sqrt{3}$

c $(2+\sqrt{2})^3$ in the form $a+b\sqrt{2}$ **d** $(1+2\sqrt{3})^4$ in the form $a+b\sqrt{3}$

6 **a** Expand $(1+x)^6$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient.

b By substituting a suitable value of x into your answer for part **a**, obtain an estimate for

i 1.02^6 **ii** 0.99^6

giving your answers to 4 decimal places.

7 **a** Expand $(1+2y)^8$ in ascending powers of y up to and including the term in y^3, simplifying each coefficient.

b By substituting a suitable value of y into your answer for part **a**, obtain an estimate for

i 0.98^8 **ii** 1.01^8

giving your answers to 4 decimal places.

8 Expand and simplify

a $(1+x)^4+(1-x)^4$ **b** $(1-\frac{1}{3}x)^3-(1+\frac{1}{3}x)^3$

9 The coefficient of x^2 in the expansion of $(1+ax)^4$ in ascending powers of x is 24, where a is a constant and $a<0$. Find

a the value of a,

b the value of the coefficient of x^3 in the expansion.

C2 SEQUENCES AND SERIES *Worksheet D*

1 Expand

a $(1 + 3x)^4$ **b** $(2 - x)^5$ **c** $(3 + 10x^2)^3$ **d** $(a + 2b)^5$

e $(x^2 - y)^3$ **f** $(5 + \frac{1}{2}x)^4$ **g** $(x + \frac{1}{x})^4$ **h** $(t - \frac{2}{t^2})^3$

2 Find the first four terms in the expansion in ascending powers of x of

a $(1 + 3x)^6$ **b** $(1 - \frac{1}{4}x)^8$ **c** $(5 - x)^7$ **d** $(3 + 2x^2)^{10}$

3 Find the coefficient indicated in the following expansions

a $(1 + x)^{15}$, coefficient of x^3 **b** $(1 - 2x)^{12}$, coefficient of x^4

c $(3 + x)^7$, coefficient of x^2 **d** $(2 - y)^{10}$, coefficient of y^5

e $(2 + t^3)^8$, coefficient of t^{15} **f** $(1 - \frac{1}{x})^9$, coefficient of x^{-3}

4 **a** Express $(\sqrt{2} - \sqrt{5})^4$ in the form $a + b\sqrt{10}$, where $a, b \in \mathbb{Z}$.

b Express $(\sqrt{2} + \frac{1}{\sqrt{3}})^3$ in the form $a\sqrt{2} + b\sqrt{3}$, where $a, b \in \mathbb{Q}$.

c Express $(1 + \sqrt{5})^3 - (1 - \sqrt{5})^3$ in the form $a\sqrt{5}$, where $a \in \mathbb{Z}$.

5 **a** Expand $(1 + \frac{x}{2})^{10}$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient.

b By substituting a suitable value of x into your answer for part **a**, obtain an estimate for

i 1.005^{10} **ii** 0.996^{10}

giving your answers to 5 decimal places.

6 **a** Expand $(3 + x)^8$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient.

b By substituting a suitable value of x into your answer for part **a**, obtain an estimate for

i 3.001^8 **ii** 2.995^8

giving your answers to 7 significant figures.

7 Expand and simplify

a $(1 + 10x)^4 + (1 - 10x)^4$ **b** $(2 - \frac{1}{3}x)^3 - (2 + \frac{1}{3}x)^3$

c $(1 + 4y)(1 + y)^3$ **d** $(1 - x)(1 + \frac{1}{x})^3$

8 Expand each of the following in ascending powers of x up to and including the term in x^3.

a $(1 + x^2)(1 - 3x)^{10}$ **b** $(1 - 2x)(1 + x)^8$

c $(1 + x + x^2)(1 - x)^6$ **d** $(1 + 3x - x^2)(1 + 2x)^7$

9 Find the term independent of y in each of the following expansions.

a $(y + \frac{1}{y})^8$ **b** $(2y - \frac{1}{2y})^{12}$ **c** $(\frac{1}{y} + y^2)^6$ **d** $(3y - \frac{1}{y^2})^9$

10 The coefficient of x^2 in the binomial expansion of $(1 + \frac{2}{5}x)^n$, where n is a positive integer, is 1.6

a Find the value of n.

b Use your value of n to find the coefficient of x^4 in the expansion.

11 Given that $y_1 = (1 - 2x)(1 + x)^{10}$ and $y_2 = ax^2 + bx + c$ and that when x is small, y_2 can be used as an approximation for y_1,

a find the values of the constants a, b and c,

b find the percentage error in using y_2 as an approximation for y_1 when $x = 0.2$

12 In the binomial expansion of $(1 + px)^q$, where p and q are constants and q is a positive integer, the coefficient of x is –12 and the coefficient of x^2 is 60.

Find

a the value of p and the value of q,

b the value of the coefficient of x^3 in the expansion.

13 **a** Expand $(3 - \frac{x}{3})^{12}$ as a binomial series in ascending powers of x up to and including the term in x^3, giving each coefficient as an integer.

b Use your series expansion with a suitable value of x to obtain an estimate for 2.998^{12}, giving your answer to 2 decimal places.

14 **a** Expand $(1 - x)^5$ as a binomial series in ascending powers of x.

b Express $(\sqrt{3} + 1)(\sqrt{3} - 2)$ in the form $A + B\sqrt{3}$, where $A, B \in \mathbb{Z}$.

c Hence express each of the following in the form $C + D\sqrt{3}$, where $C, D \in \mathbb{Z}$.

i $(\sqrt{3} + 1)^5(\sqrt{3} - 2)^5$

ii $(\sqrt{3} + 1)^6(\sqrt{3} - 2)^5$

15 **a** Expand $(1 + \frac{x}{2})^9$ in ascending powers of x up to and including the term in x^4.

Hence, or otherwise, find

b the coefficient of x^3 in the expansion of $(1 + \frac{x}{2})^9 - (1 - \frac{x}{2})^9$,

c the coefficient of x^4 in the expansion of $(1 + 2x)(1 + \frac{x}{2})^9$.

16 The term independent of x in the expansion of $(x^3 + \frac{a}{x^2})^5$ is –80.

Find the value of the constant a.

17 In the binomial expansion of $(1 + \frac{x}{k})^n$, where k is a non-zero constant, n is an integer and $n > 1$, the coefficient of x^2 is three times the coefficient of x^3.

a Show that $k = n - 2$.

Given also that $n = 7$,

b expand $(1 + \frac{x}{k})^n$ in ascending powers of x up to and including the term in x^4, giving each coefficient as a fraction in its simplest form.

C2 SEQUENCES AND SERIES *Worksheet E*

1 Expand $(1 + 4x)^4$ in ascending powers of x, simplifying the coefficients. **(4)**

2 A geometric series has first term 3 and common ratio –2.

a Find the fifth term of the series. **(2)**

b Find the sum of the first ten terms of the series. **(2)**

c Show that the sum of the first eight positive terms of the series is 65 535. **(4)**

3 **a** Expand $(1 + 3x)^7$ in ascending powers of x up to and including the term in x^4, simplifying each coefficient in the expansion. **(4)**

b Use your series with a suitable value of x to estimate the value of 1.03^7 correct to 5 decimal places. **(3)**

4 Evaluate $\sum_{r=3}^{12} 2^r$. **(4)**

5 **a** Expand $(2 + x)^5$, simplifying the coefficient in each term. **(4)**

b Hence, or otherwise, write down the expansion of $(2 - x)^5$. **(1)**

c Show that

$$(2 + \sqrt{5})^5 - (2 - \sqrt{5})^5 = k\sqrt{5},$$

where k is an integer to be found. **(4)**

6 Ginny opens a savings account and decides to pay £200 into the account at the start of each month. At the end of each month, interest of 0.5% is paid into the account.

a Find, to the nearest penny, the interest paid into the account at the end of the third month. **(4)**

b Show that the total interest paid into the account over the first 12 months is £79.45 to the nearest penny. **(5)**

7 Find the first four terms in the expansion of $(1 - 3x)^8$ in ascending powers of x, simplifying each coefficient. **(4)**

8 **a** Prove that the sum, S_n, of the first n terms of a geometric series with first term a and common ratio r is given by

$$S_n = \frac{a(1-r^n)}{1-r}.$$ **(4)**

b Find the exact sum of the first 16 terms of the geometric series with fourth term 3 and fifth term 6. **(5)**

9 **a** Write down the first three terms in the binomial expansion of $(1 + ax)^n$, where n is a positive integer, in ascending powers of x. **(2)**

Given that the coefficient of x^2 is three times the coefficient of x,

b show that $n = \frac{6+a}{a}$. **(4)**

Given also that $a = \frac{2}{3}$,

c find the coefficient of x^3 in the expansion. **(3)**

10 Find the first three terms in the expansion of $(2 + 5x)^6$ in ascending powers of x, simplifying each coefficient. **(4)**

11 The first term of a geometric series is 162 and the sum to infinity of the series is 486.

a Find the common ratio of the series. **(3)**

b Find the sixth term of the series. **(2)**

c Find, to 3 decimal places, the sum of the first ten terms of the series. **(4)**

12 **a** Expand $(1 + 3x)^4$ in ascending powers of x, simplifying the coefficients. **(4)**

b Find the coefficient of x^2 in the expansion of

$$(1 + 4x - x^2)(1 + 3x)^4.$$ **(3)**

13 In a computer game, each player must complete the tasks set at each level within a fixed amount of time in order to progress to the next level.

The time allowed for level 1 is 2 minutes and the time allowed for each of the other levels is 10% less than that allowed in the previous level.

a Find, in seconds, the time allowed for completing level 4. **(2)**

b Find, in minutes and seconds, the maximum total time allowed for completing the first 12 levels of the game. **(4)**

14 Given that

$$(1 + \tfrac{x}{2})^8(1 - x)^6 \equiv 1 + Ax + Bx^2 + \ldots,$$

find the values of the constants A and B. **(7)**

15 The terms of a sequence are defined by the recurrence relation

$$u_r = 2u_{r-1}, \quad r > 1, \quad u_1 = 6.$$

a Write down the first four terms of the sequence. **(1)**

b Evaluate $\sum_{r=1}^{10} u_r$. **(3)**

16 **a** Expand $(1 + x)^4$ in ascending powers of x. **(2)**

b Hence, or otherwise, write down the expansion of $(1 - x)^4$ in ascending powers of x. **(1)**

c By using your answers to parts **a** and **b**, or otherwise, solve the equation

$$(1 + x)^4 + (1 - x)^4 = 82,$$

for real values of x. **(5)**

17 The common ratio of a geometric series is 1.5 and the third term of the series is 18.

a Find the first term of the series. **(2)**

b Find the sum of the first six terms of the series. **(2)**

c Find the smallest value of k such that the kth term of the series is greater than 8000. **(4)**

18 The first two terms in the expansion of $(1 + \tfrac{ax}{2})^{10} + (1 + bx)^{10}$, in ascending powers of x, are 2 and $90x^2$.

Given that $a < b$, find the values of the constants a and b. **(9)**