

C2 SEQUENCES AND SERIES

Answers - Worksheet C

- 1**
- a** $= 1 + 4x + 6x^2 + 4x^3 + x^4$ **b** $= 1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5$
- c** $= 1 + 3(4x) + 3(4x)^2 + (4x)^3$ **d** $= 1 + 3(-2y) + 3(-2y)^2 + (-2y)^3$
 $= 1 + 12x + 48x^2 + 64x^3$ $= 1 - 6y + 12y^2 - 8y^3$
- e** $= 1 + 4(\frac{1}{2}x) + 6(\frac{1}{2}x)^2 + 4(\frac{1}{2}x)^3 + (\frac{1}{2}x)^4$ **f** $= 1 + 3(\frac{1}{3}y) + 3(\frac{1}{3}y)^2 + (\frac{1}{3}y)^3$
 $= 1 + 2x + \frac{3}{2}x^2 + \frac{1}{2}x^3 + \frac{1}{16}x^4$ $= 1 + y + \frac{1}{3}y^2 + \frac{1}{27}y^3$
- g** $= 1 + 5(x^2) + 10(x^2)^2 + 10(x^2)^3 + 5(x^2)^4 + (x^2)^5$ **h** $= 1 + 4(-\frac{3}{2}x) + 6(-\frac{3}{2}x)^2 + 4(-\frac{3}{2}x)^3 + (-\frac{3}{2}x)^4$
 $= 1 + 5x^2 + 10x^4 + 10x^6 + 5x^8 + x^{10}$ $= 1 - 6x + \frac{27}{2}x^2 - \frac{27}{2}x^3 + \frac{81}{16}x^4$
- 2**
- a** $= x^3 + 3x^2y + 3xy^2 + y^3$ **b** $= a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5$
- c** $= x^4 + 4x^3(2y) + 6x^2(2y)^2 + 4x(2y)^3 + (2y)^4$ **d** $= 2^3 + 3(2^2)y + 3(2)y^2 + y^3$
 $= x^4 + 8x^3y + 24x^2y^2 + 32xy^3 + 16y^4$ $= 8 + 12y + 6y^2 + y^3$
- e** $= 3^3 + 3(3^2)(-x) + 3(3)(-x)^2 + (-x)^3$ **f** $= 5^4 + 4(5^3)(2x) + 6(5^2)(2x)^2 + 4(5)(2x)^3 + (2x)^4$
 $= 27 - 27x + 9x^2 - x^3$ $= 625 + 1000x + 600x^2 + 160x^3 + 16x^4$
- g** $= 3^5 + 5(3^4)(-4y) + 10(3^3)(-4y)^2 + 10(3^2)(-4y)^3 + 5(3)(-4y)^4 + (-4y)^5$
 $= 243 - 1620y + 4320y^2 - 5760y^3 + 3840y^4 - 1024y^5$
- h** $= 3^4 + 4(3^3)(\frac{1}{2}x) + 6(3^2)(\frac{1}{2}x)^2 + 4(3)(\frac{1}{2}x)^3 + (\frac{1}{2}x)^4$
 $= 81 + 54x + \frac{27}{2}x^2 + \frac{3}{2}x^3 + \frac{1}{16}x^4$
- 3**
- a** $= 1 + 10x + \frac{10 \times 9}{2}x^2 + \frac{10 \times 9 \times 8}{3 \times 2}x^3 + \dots$ **b** $= 1 + 6(-x) + \frac{6 \times 5}{2}(-x)^2 + \frac{6 \times 5 \times 4}{3 \times 2}(-x)^3 + \dots$
 $= 1 + 10x + 45x^2 + 120x^3 + \dots$ $= 1 - 6x + 15x^2 - 20x^3 + \dots$
- c** $= 1 + 8(2x) + \frac{8 \times 7}{2}(2x)^2 + \frac{8 \times 7 \times 6}{3 \times 2}(2x)^3 + \dots$ **d** $= 1 + 7(-\frac{1}{2}x) + \frac{7 \times 6}{2}(-\frac{1}{2}x)^2 + \frac{7 \times 6 \times 5}{3 \times 2}(-\frac{1}{2}x)^3 + \dots$
 $= 1 + 16x + 112x^2 + 448x^3 + \dots$ $= 1 - \frac{7}{2}x + \frac{21}{4}x^2 - \frac{35}{8}x^3 + \dots$
- e** $= 1 + 6(x^3) + \frac{6 \times 5}{2}(x^3)^2 + \frac{6 \times 5 \times 4}{3 \times 2}(x^3)^3 + \dots$ **f** $= 2^9 + 9(2^8)x + \frac{9 \times 8}{2}(2^7)x^2 + \frac{9 \times 8 \times 7}{3 \times 2}(2^6)x^3 + \dots$
 $= 1 + 6x^3 + 15x^6 + 20x^9 + \dots$ $= 512 + 2304x + 4608x^2 + 5376x^3 + \dots$
- g** $= 3^7 + 7(3^6)(-x) + \frac{7 \times 6}{2}(3^5)(-x)^2 + \frac{7 \times 6 \times 5}{3 \times 2}(3^4)(-x)^3 + \dots$
 $= 2187 - 5103x + 5103x^2 - 2835x^3 + \dots$
- h** $= 2^{10} + 10(2^9)(5x) + \frac{10 \times 9}{2}(2^8)(5x)^2 + \frac{10 \times 9 \times 8}{3 \times 2}(2^7)(5x)^3 + \dots$
 $= 1024 + 25\,600x + 288\,000x^2 + 1\,920\,000x^3 + \dots$
- 4**
- a** $= \binom{20}{3} = 1140$ **b** $= \binom{14}{4} \times (-1)^4 = 1001$
- c** $= \binom{9}{2} \times 4^2 = 576$ **d** $= \binom{14}{3} \times (-3)^3 = -9828$
- e** $= \binom{12}{4} \times (-\frac{1}{3})^4 = \frac{55}{9}$ or $6\frac{1}{9}$ **f** $= \binom{16}{5} \times (-\frac{1}{2})^5 = -136.5$
- g** $= \binom{15}{2} \times (\frac{2}{5})^2 = \frac{84}{5}$ or 16.8 **h** $= \binom{8}{3} = 56$

- 5 **a** $= 1 + 3(\sqrt{5}) + 3(\sqrt{5})^2 + (\sqrt{5})^3$
 $= 1 + 3\sqrt{5} + 15 + 5\sqrt{5}$
 $= 16 + 8\sqrt{5}$
- b** $= 1 + 4(-\sqrt{3}) + 6(-\sqrt{3})^2 + 4(-\sqrt{3})^3 + (-\sqrt{3})^4$
 $= 1 - 4\sqrt{3} + 18 - 12\sqrt{3} + 9$
 $= 28 - 16\sqrt{3}$
- c** $= 2^3 + 3(2^2)(\sqrt{2}) + 3(2)(\sqrt{2})^2 + (\sqrt{2})^3$
 $= 8 + 12\sqrt{2} + 12 + 2\sqrt{2}$
 $= 20 + 14\sqrt{2}$
- d** $= 1 + 4(2\sqrt{3}) + 6(2\sqrt{3})^2 + 4(2\sqrt{3})^3 + (2\sqrt{3})^4$
 $= 1 + 8\sqrt{3} + 72 + 96\sqrt{3} + 144$
 $= 217 + 104\sqrt{3}$
- 6 **a** $= 1 + 6x + \frac{6 \times 5}{2} x^2 + \frac{6 \times 5 \times 4}{3 \times 2} x^3 + \dots$
 $= 1 + 6x + 15x^2 + 20x^3 + \dots$
- b i** let $x = 0.02$
 $1.02^6 \approx 1 + 6(0.02) + 15(0.02)^2 + 20(0.02)^3$
 $= 1 + 0.12 + 0.0060 + 0.000160$
 $= 1.1262$ (4dp)
- ii** let $x = -0.01$
 $0.99^6 \approx 1 + 6(-0.01) + 15(-0.01)^2 + 20(-0.01)^3$
 $= 1 - 0.06 + 0.0015 - 0.00020$
 $= 0.9415$ (4dp)
- 7 **a** $= 1 + 8(2y) + \frac{8 \times 7}{2} (2y)^2 + \frac{8 \times 7 \times 6}{3 \times 2} (2y)^3 + \dots$
 $= 1 + 16y + 112y^2 + 448y^3 + \dots$
- b i** let $y = -0.01$
 $0.98^8 \approx 1 + 16(-0.01) + 112(-0.01)^2 + 448(-0.01)^3$
 $= 1 - 0.16 + 0.0112 - 0.000448$
 $= 0.8508$ (4dp)
- ii** let $y = 0.005$
 $1.01^8 \approx 1 + 16(0.005) + 112(0.005)^2 + 448(0.005)^3$
 $= 1 + 0.080 + 0.002800 + 0.000056000$
 $= 1.0829$ (4dp)
- 8 **a** $= 1 + 4x + 6x^2 + 4x^3 + x^4 + (1 - 4x + 6x^2 - 4x^3 + x^4)$
 $= 2 + 12x^2 + 2x^4$
- b** $= 1 + 3(-\frac{1}{3}x) + 3(-\frac{1}{3}x)^2 + (-\frac{1}{3}x)^3 - [1 + 3(\frac{1}{3}x) + 3(\frac{1}{3}x)^2 + (\frac{1}{3}x)^3]$
 $= 1 - x + \frac{1}{3}x^2 - \frac{1}{27}x^3 - (1 + x + \frac{1}{3}x^2 + \frac{1}{27}x^3)$
 $= -2x - \frac{2}{27}x^3$
- 9 **a** $6(ax)^2 = 24x^2$
 $a^2 = 4$
 $a < 0 \therefore a = -2$
- b** $= 4a^3 = -32$

C2 SEQUENCES AND SERIES

Answers - Worksheet D

- 1**
- a** $= 1 + 4(3x) + 6(3x)^2 + 4(3x)^3 + (3x)^4$
 $= 1 + 12x + 54x^2 + 108x^3 + 81x^4$
- b** $= 2^5 + 5(2^4)(-x) + 10(2^3)(-x)^2 + 10(2^2)(-x)^3 + 5(2)(-x)^4 + (-x)^5$
 $= 32 - 80x + 80x^2 - 40x^3 + 10x^4 - x^5$
- c** $= 3^3 + 3(3^2)(10x^2) + 3(3)(10x^2)^2 + (10x^2)^3$
 $= 27 + 270x^2 + 900x^4 + 1000x^6$
- d** $= a^5 + 5a^4(2b) + 10a^3(2b)^2 + 10a^2(2b)^3 + 5a(2b)^4 + (2b)^5$
 $= a^5 + 10a^4b + 40a^3b^2 + 80a^2b^3 + 80ab^4 + 32b^5$
- e** $= (x^2)^3 + 3(x^2)^2(-y) + 3(x^2)(-y)^2 + (-y)^3$
 $= x^6 - 3x^4y + 3x^2y^2 - y^3$
- f** $= 5^4 + 4(5^3)(\frac{1}{2}x) + 6(5^2)(\frac{1}{2}x)^2 + 4(5)(\frac{1}{2}x)^3 + (\frac{1}{2}x)^4$
 $= 625 + 250x + \frac{75}{2}x^2 + \frac{5}{2}x^3 + \frac{1}{16}x^4$
- g** $= x^4 + 4x^3(\frac{1}{x}) + 6x^2(\frac{1}{x})^2 + 4x(\frac{1}{x})^3 + (\frac{1}{x})^4$
 $= x^4 + 4x^2 + 6 + \frac{4}{x^2} + \frac{1}{x^4}$
- h** $= t^3 + 3t^2(-\frac{2}{t^2}) + 3t(-\frac{2}{t^2})^2 + (-\frac{2}{t^2})^3$
 $= t^3 - 6 + \frac{12}{t^3} - \frac{8}{t^6}$
- 2**
- a** $= 1 + 6(3x) + \frac{6 \times 5}{2}(3x)^2 + \frac{6 \times 5 \times 4}{3 \times 2}(3x)^3 + \dots$
 $= 1 + 18x + 135x^2 + 540x^3 + \dots$
- b** $= 1 + 8(-\frac{1}{4}x) + \frac{8 \times 7}{2}(-\frac{1}{4}x)^2 + \frac{8 \times 7 \times 6}{3 \times 2}(-\frac{1}{4}x)^3 + \dots$
 $= 1 - 2x + \frac{7}{4}x^2 - \frac{7}{8}x^3 + \dots$
- c** $= 5^7 + 7(5^6)(-x) + \frac{7 \times 6}{2}(5^5)(-x)^2 + \frac{7 \times 6 \times 5}{3 \times 2}(5^4)(-x)^3 + \dots$
 $= 78\,125 - 109\,375x + 65\,625x^2 - 21\,875x^3 + \dots$
- d** $= 3^{10} + 10(3^9)(2x^2) + \frac{10 \times 9}{2}(3^8)(2x^2)^2 + \frac{10 \times 9 \times 8}{3 \times 2}(3^7)(2x^2)^3 + \dots$
 $= 59\,049 + 393\,660x^2 + 1\,180\,980x^4 + 2\,099\,520x^6 + \dots$
- 3**
- a** $= \binom{15}{3} = 455$
- b** $= \binom{12}{4} \times (-2)^4 = 7920$
- c** $= \binom{7}{2} \times 3^5 = 5103$
- d** $= \binom{10}{5} \times 2^5 \times (-1)^5 = -8064$
- e** $= \binom{8}{5} \times 2^3 = 448$
- f** $= \binom{9}{3} \times (-1)^3 = -84$
- 4**
- a** $= (\sqrt{2})^4 + 4(\sqrt{2})^3(-\sqrt{5}) + 6(\sqrt{2})^2(-\sqrt{5})^2 + 4(\sqrt{2})(-\sqrt{5})^3 + (-\sqrt{5})^4$
 $= 4 - 8\sqrt{10} + 60 - 20\sqrt{10} + 25$
 $= 89 - 28\sqrt{10}$
- b** $= (\sqrt{2})^3 + 3(\sqrt{2})^2(\frac{1}{\sqrt{3}}) + 3(\sqrt{2})(\frac{1}{\sqrt{3}})^2 + (\frac{1}{\sqrt{3}})^3$
 $= 2\sqrt{2} + 2\sqrt{3} + \sqrt{2} + \frac{1}{9}\sqrt{3}$
 $= 3\sqrt{2} + \frac{19}{9}\sqrt{3}$

$$\begin{aligned}
 \mathbf{c} &= 1 + 3(\sqrt{5}) + 3(\sqrt{5})^2 + (\sqrt{5})^3 - [1 + 3(-\sqrt{5}) + 3(-\sqrt{5})^2 + (-\sqrt{5})^3] \\
 &= 1 + 3\sqrt{5} + 15 + 5\sqrt{5} - [1 - 3\sqrt{5} + 15 - 5\sqrt{5}] \\
 &= 16 + 8\sqrt{5} - [16 - 8\sqrt{5}] \\
 &= 16\sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{5} \quad \mathbf{a} &= 1 + 10\left(\frac{x}{2}\right) + \frac{10 \times 9}{2} \left(\frac{x}{2}\right)^2 + \frac{10 \times 9 \times 8}{3 \times 2} \left(\frac{x}{2}\right)^3 + \dots \\
 &= 1 + 5x + \frac{45}{4}x^2 + 15x^3 + \dots
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad \mathbf{i} \quad \text{let } x &= 0.01 \\
 1.005^{10} &\approx 1 + 0.05 + 0.001125 + 0.000015 \\
 &= 1.05114 \text{ (5dp)}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{ii} \quad \text{let } x &= -0.008 \\
 0.996^{10} &\approx 1 - 0.040 + 0.000720 - 0.000007680 \\
 &= 0.96071 \text{ (5dp)}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{6} \quad \mathbf{a} &= 3^8 + 8(3^7)x + \frac{8 \times 7}{2} (3^6)x^2 + \frac{8 \times 7 \times 6}{3 \times 2} (3^5)x^3 + \dots \\
 &= 6561 + 17496x + 20412x^2 + 13608x^3 + \dots
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad \mathbf{i} \quad \text{let } x &= 0.001 \\
 3.001^8 &\approx 6561 + 17.496 + 0.020412 + 0.000013608 \\
 &= 6578.516 \text{ (7sf)}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{ii} \quad \text{let } x &= -0.005 \\
 2.995^8 &\approx 6561 - 87.480 + 0.510300 - 0.001701000 \\
 &= 6474.029 \text{ (7sf)}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{7} \quad \mathbf{a} \quad (1 + 10x)^4 &= 1 + 4(10x) + 6(10x)^2 + 4(10x)^3 + (10x)^4 \\
 &= 1 + 40x + 600x^2 + 4000x^3 + 10000x^4 \\
 \therefore (1 + 10x)^4 + (1 - 10x)^4 &= 1 + 40x + 600x^2 + 4000x^3 + 10000x^4 + (1 - 40x + 600x^2 - 4000x^3 + 10000x^4) \\
 &= 2 + 1200x^2 + 20000x^4
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad (2 + \frac{1}{3}x)^3 &= 2^3 + 3(2^2)(\frac{1}{3}x) + 3(2)(\frac{1}{3}x)^2 + (\frac{1}{3}x)^3 \\
 &= 8 + 4x + \frac{2}{3}x^2 + \frac{1}{27}x^3 \\
 \therefore (2 - \frac{1}{3}x)^3 - (2 + \frac{1}{3}x)^3 &= 8 - 4x + \frac{2}{3}x^2 - \frac{1}{27}x^3 - (8 + 4x + \frac{2}{3}x^2 + \frac{1}{27}x^3) \\
 &= -8x - \frac{2}{27}x^3
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} &= (1 + 4y)(1 + 3y + 3y^2 + y^3) \\
 &= 1 + 3y + 3y^2 + y^3 + 4y + 12y^2 + 12y^3 + 4y^4 \\
 &= 1 + 7y + 15y^2 + 13y^3 + 4y^4
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{d} &= (1 - x)\left(1 + \frac{3}{x} + \frac{3}{x^2} + \frac{1}{x^3}\right) \\
 &= 1 + \frac{3}{x} + \frac{3}{x^2} + \frac{1}{x^3} - x - 3 - \frac{3}{x} - \frac{1}{x^2} \\
 &= -x - 2 + \frac{2}{x^2} + \frac{1}{x^3}
 \end{aligned}$$

$$\begin{aligned}
 8 \quad \mathbf{a} &= (1+x^2)\left[1+10(-3x)+\frac{10 \times 9}{2}(-3x)^2+\frac{10 \times 9 \times 8}{3 \times 2}(-3x)^3+\dots\right] \\
 &= (1+x^2)[1-30x+405x^2-3240x^3+\dots] \\
 &= 1-30x+405x^2-3240x^3+x^2-30x^3+\dots \\
 &= 1-30x+406x^2-3270x^3+\dots
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} &= (1-2x)\left[1+8x+\frac{8 \times 7}{2}x^2+\frac{8 \times 7 \times 6}{3 \times 2}x^3+\dots\right] \\
 &= (1-2x)[1+8x+28x^2+56x^3+\dots] \\
 &= 1+8x+28x^2+56x^3-2x-16x^2-56x^3+\dots \\
 &= 1+6x+12x^2+\dots
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} &= (1+x+x^2)\left[1+6(-x)+\frac{6 \times 5}{2}(-x)^2+\frac{6 \times 5 \times 4}{3 \times 2}(-x)^3+\dots\right] \\
 &= (1+x+x^2)[1-6x+15x^2-20x^3+\dots] \\
 &= 1-6x+15x^2-20x^3+x-6x^2+15x^3+x^2-6x^3+\dots \\
 &= 1-5x+10x^2-11x^3+\dots
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{d} &= (1+3x-x^2)\left[1+7(2x)+\frac{7 \times 6}{2}(2x)^2+\frac{7 \times 6 \times 5}{3 \times 2}(2x)^3+\dots\right] \\
 &= (1+3x-x^2)[1+14x+84x^2+280x^3+\dots] \\
 &= 1+14x+84x^2+280x^3+3x+42x^2+252x^3-x^2-14x^3+\dots \\
 &= 1+17x+125x^2+518x^3+\dots
 \end{aligned}$$

$$9 \quad \mathbf{a} = \binom{8}{4} \times y^4 \times \left(\frac{1}{y}\right)^4 = 70$$

$$\mathbf{b} = \binom{12}{6} \times (2y)^6 \times \left(-\frac{1}{2y}\right)^6 = 924$$

$$\mathbf{c} = \binom{6}{2} \times \left(\frac{1}{y}\right)^4 \times (y^2)^2 = 15$$

$$\mathbf{d} = \binom{9}{3} \times (3y)^6 \times \left(-\frac{1}{y^2}\right)^3 = -61\,236$$

$$10 \quad \mathbf{a} \quad \frac{n(n-1)}{2} \times \left(\frac{2}{5}\right)^2 = 1.6$$

$$n(n-1) = \frac{25}{2} \times 1.6 = 20$$

$$n^2 - n - 20 = 0$$

$$(n+4)(n-5) = 0$$

$$n > 0 \quad \therefore n = 5$$

$$\mathbf{b} = 5 \times \left(\frac{2}{5}\right)^4 = \frac{16}{125} \text{ or } 0.128$$

$$\begin{aligned}
 11 \quad \mathbf{a} \quad y_1 &= (1-2x)\left[1+10x+\frac{10 \times 9}{2}x^2+\dots\right] \\
 &= 1+10x+45x^2-2x-20x^2+\dots \\
 &= 1+8x+25x^2+\dots
 \end{aligned}$$

$$\therefore a = 25, b = 8, c = 1$$

$$\mathbf{b} \quad x = 0.2: \quad y_1 = 0.6 \times (1.2)^{10} = 3.71504$$

$$y_2 = (25 \times 0.04) + (8 \times 0.2) + 1 = 3.6$$

$$\% \text{ error} = \frac{3.71504 - 3.6}{3.71504} \times 100\% = 3.1\% \text{ (2sf)}$$

$$12 \quad \mathbf{a} \quad (1+px)^q = 1 + q(px) + \frac{q(q-1)}{2}(px)^2 + \dots$$

$$\therefore pq = -12 \text{ and } \frac{1}{2}p^2q(q-1) = 60$$

$$\text{sub. } p = -\frac{12}{q}$$

$$\Rightarrow \frac{72}{q}(q-1) = 60$$

$$72(q-1) = 60q$$

$$q = 6, p = -2$$

$$\mathbf{b} = \frac{6 \times 5 \times 4}{3 \times 2} \times (-2)^3 = -160$$

$$13 \quad \mathbf{a} = 3^{12} + 12(3^{11})\left(-\frac{x}{3}\right) + \frac{12 \times 11}{2} (3^{10})\left(-\frac{x}{3}\right)^2 + \frac{12 \times 11 \times 10}{3 \times 2} (3^9)\left(-\frac{x}{3}\right)^3 + \dots$$

$$= 531\,441 - 708\,588x + 433\,026x^2 - 160\,380x^3 + \dots$$

$$\mathbf{b} \text{ let } \frac{x}{3} = 0.002 \quad \therefore x = 0.006$$

$$2.998^{12} \approx 531\,441 - 4251.528 + 15.588\,936 - 0.034\,642\,080$$

$$= 527\,205.03 \text{ (2dp)}$$

$$14 \quad \mathbf{a} = 1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5$$

$$\mathbf{b} = 3 - 2\sqrt{3} + \sqrt{3} - 2 = 1 - \sqrt{3}$$

$$\mathbf{c} \quad \mathbf{i} = [(\sqrt{3} + 1)(\sqrt{3} - 2)]^5 = (1 - \sqrt{3})^5$$

$$= 1 - 5(\sqrt{3}) + 10(\sqrt{3})^2 - 10(\sqrt{3})^3 + 5(\sqrt{3})^4 - (\sqrt{3})^5$$

$$= 1 - 5\sqrt{3} + 30 - 30\sqrt{3} + 45 - 9\sqrt{3}$$

$$= 76 - 44\sqrt{3}$$

$$\mathbf{ii} = (\sqrt{3} + 1)(76 - 44\sqrt{3})$$

$$= 76\sqrt{3} - 132 + 76 - 44\sqrt{3}$$

$$= -56 + 32\sqrt{3}$$

$$15 \quad \mathbf{a} = 1 + 9\left(\frac{x}{2}\right) + \frac{9 \times 8}{2} \left(\frac{x}{2}\right)^2 + \frac{9 \times 8 \times 7}{3 \times 2} \left(\frac{x}{2}\right)^3 + \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2} \left(\frac{x}{2}\right)^4 + \dots$$

$$= 1 + \frac{9}{2}x + 9x^2 + \frac{21}{2}x^3 + \frac{63}{8}x^4 + \dots$$

$$\mathbf{b} = \frac{21}{2} - \left(-\frac{21}{2}\right) = 21$$

$$\mathbf{c} = \left(1 \times \frac{63}{8}\right) + \left(2 \times \frac{21}{2}\right) = 28\frac{7}{8}$$

$$16 \quad 10(x^3)^2 \left(\frac{a}{x^2}\right)^3 = -80$$

$$a^3 = -8$$

$$a = -2$$

$$17 \quad \mathbf{a} \quad \left(1 + \frac{x}{k}\right)^n = 1 + n\left(\frac{x}{k}\right) + \frac{n(n-1)}{2} \left(\frac{x}{k}\right)^2 + \frac{n(n-1)(n-2)}{3 \times 2} \left(\frac{x}{k}\right)^3 + \dots$$

$$\therefore \frac{n(n-1)}{2k^2} = 3 \times \frac{n(n-1)(n-2)}{6k^3}$$

$$kn(n-1) = n(n-1)(n-2)$$

$$n(n-1)[k - (n-2)] = 0$$

$$n > 1 \quad \therefore k - (n-2) = 0$$

$$k = n - 2$$

$$\mathbf{b} \quad k = 7 - 2 = 5$$

$$\left(1 + \frac{x}{5}\right)^7 = 1 + 7\left(\frac{x}{5}\right) + \frac{7 \times 6}{2} \left(\frac{x}{5}\right)^2 + \frac{7 \times 6 \times 5}{3 \times 2} \left(\frac{x}{5}\right)^3 + \frac{7 \times 6 \times 5 \times 4}{4 \times 3 \times 2} \left(\frac{x}{5}\right)^4 + \dots$$

$$= 1 + \frac{7}{5}x + \frac{21}{25}x^2 + \frac{7}{25}x^3 + \frac{7}{125}x^4 + \dots$$

C2 SEQUENCES AND SERIES

Answers - Worksheet E

$$1 \quad = 1 + 4(4x) + 6(4x)^2 + 4(4x)^3 + (4x)^4 \\ = 1 + 16x + 96x^2 + 256x^3 + 256x^4$$

$$2 \quad \mathbf{a} \quad u_5 = 3 \times (-2)^4 = 48$$

$$\mathbf{b} \quad S_{10} = \frac{3[1 - (-2)^{10}]}{1 - (-2)} = -1023$$

\mathbf{c} positive terms form GP:

$$a = 3, r = (-2)^2 = 4$$

$$S_8 = \frac{3(4^8 - 1)}{4 - 1} = 65\,535$$

$$3 \quad \mathbf{a} \quad = 1 + 7(3x) + \frac{7 \times 6}{2} (3x)^2 \\ + \frac{7 \times 6 \times 5}{3 \times 2} (3x)^3 + \frac{7 \times 6 \times 5 \times 4}{4 \times 3 \times 2} (3x)^4 + \dots \\ = 1 + 21x + 189x^2 + 945x^3 + 2835x^4 + \dots$$

\mathbf{b} let $x = 0.01$

$$1.03^7 \approx 1 + 0.21 + 0.0189 \\ + 0.000\,945 + 0.000\,028\,35 \\ = 1.229\,87 \text{ (5dp)}$$

$$4 \quad \text{GP: } a = 8, r = 2, n = 10$$

$$S_{10} = \frac{8(2^{10} - 1)}{2 - 1} = 8184$$

$$5 \quad \mathbf{a} \quad = 2^5 + 5(2^4)x + 10(2^3)x^2 \\ + 10(2^2)x^3 + 5(2)x^4 + x^5 \\ = 32 + 80x + 80x^2 + 40x^3 + 10x^4 + x^5 \\ \mathbf{b} \quad = 32 - 80x + 80x^2 - 40x^3 + 10x^4 - x^5 \\ \mathbf{c} \quad (2 + \sqrt{5})^5 = 32 + 80(\sqrt{5}) + 80(\sqrt{5})^2 \\ + 40(\sqrt{5})^3 + 10(\sqrt{5})^4 + (\sqrt{5})^5 \\ = 32 + 80\sqrt{5} + 400 + 200\sqrt{5} + 250 + 25\sqrt{5} \\ = 682 + 305\sqrt{5} \\ \therefore (2 + \sqrt{5})^5 - (2 - \sqrt{5})^5 \\ = (682 + 305\sqrt{5}) - (682 - 305\sqrt{5}) \\ = 610\sqrt{5}, k = 610$$

$$6 \quad \mathbf{a} \quad \text{amount in account after } 3^{\text{rd}} \text{ payment in} \\ = 200 + (1.005 \times 200) + (1.005^2 \times 200) \\ = 603.005$$

interest paid at end of 3rd month

$$= 0.005 \times 603.005 = \text{£}3.02 \text{ (nearest penny)}$$

$$\mathbf{b} \quad \text{amount paid in} = 12 \times 200 = \text{£}2400$$

amount in account after 12 months

$$= 200(1.005 + 1.005^2 + \dots + 1.005^{12}) \\ = 200 \times S_{12} \text{ [GP: } a = 1.005, r = 1.005]$$

$$= 200 \times \frac{1.005(1.005^{12} - 1)}{1.005 - 1} = 2479.45$$

$$\text{total interest} = 2479.45 - 2400 = \text{£}79.45$$

$$7 \quad = 1 + 8(-3x) + \frac{8 \times 7}{2} (-3x)^2 \\ + \frac{8 \times 7 \times 6}{3 \times 2} (-3x)^3 + \dots \\ = 1 - 24x + 252x^2 - 1512x^3 + \dots$$

$$8 \quad \mathbf{a} \quad S_n = a + ar + ar^2 + \dots + ar^{n-1}$$

$$rS_n = ar + ar^2 + \dots + ar^{n-1} + ar^n$$

subtracting, $S_n - rS_n = a - ar^n$

$$S_n(1 - r) = a(1 - r^n)$$

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$\mathbf{b} \quad r = 6 \div 3 = 2$$

$$a \times 2^3 = 3 \quad \therefore a = \frac{3}{8}$$

$$S_{16} = \frac{\frac{3}{8}(2^{16} - 1)}{2 - 1} = 24\,575\frac{5}{8}$$

- 9** a $= 1 + n(ax) + \frac{n(n-1)}{2}(ax)^2 + \dots$
 $= 1 + anx + \frac{1}{2}a^2n(n-1)x^2 + \dots$
 b $\frac{1}{2}a^2n(n-1) = 3an$
 $a^2n(n-1) = 6an$
 $an[a(n-1) - 6] = 0$
 $n \neq 0 \therefore a(n-1) - 6 = 0$
 $an - a = 6$
 $n = \frac{6+a}{a}$
 c $n = 10 \therefore \text{coeff. of } x^3 = \frac{10 \times 9 \times 8}{3 \times 2} \times \left(\frac{2}{3}\right)^3 = 35\frac{5}{9}$
- 10** $= 2^6 + 6(2^5)(5x) + \frac{6 \times 5}{2}(2^4)(5x)^2 + \dots$
 $= 64 + 960x + 6000x^2 + \dots$
- 11** a $\frac{162}{1-r} = 486$
 $1-r = \frac{162}{486} = \frac{1}{3} \therefore r = \frac{2}{3}$
 b $u_6 = 162 \times \left(\frac{2}{3}\right)^5 = \frac{64}{3}$ or $21\frac{1}{3}$
 c $S_{10} = \frac{162[1 - (\frac{2}{3})^{10}]}{1 - \frac{2}{3}} = 477.572$
- 12** a $= 1 + 4(3x) + 6(3x)^2 + 4(3x)^3 + (3x)^4$
 $= 1 + 12x + 54x^2 + 108x^3 + 81x^4$
 b term in $x^2 = (1)(54x^2) + (4x)(12x) + (-x^2)(1)$
 coefficient of $x^2 = 54 + 48 - 1 = 101$
- 13** a time $= 120 \times (0.9)^3 = 87.48$ seconds
 b GP: $a = 120, r = 0.9, n = 12$
 $S_{12} = \frac{120[1 - (0.9)^{12}]}{1 - 0.9}$
 $= 861.08$ seconds
 $= 14$ mins 21 secs (nearest sec.)
- 14** $= [1 + 8(\frac{x}{2}) + \frac{8 \times 7}{2}(\frac{x}{2})^2 + \dots][1 + 6(-x) + \frac{6 \times 5}{2}(-x)^2 + \dots]$
 $= [1 + 4x + 7x^2 + \dots][1 - 6x + 15x^2 + \dots]$
 $= 1 - 6x + 15x^2 + 4x - 24x^2 + 7x^2 + \dots$
 $= 1 - 2x - 2x^2 + \dots$
 $\therefore A = -2, B = -2$
- 15** a 6, 12, 24, 48
 b GP: $a = 6, r = 2, n = 10$
 $S_{10} = \frac{6(2^{10} - 1)}{2 - 1} = 6138$
- 16** a $= 1 + 4x + 6x^2 + 4x^3 + x^4$
 b $= 1 - 4x + 6x^2 - 4x^3 + x^4$
 c $(1 + 4x + 6x^2 + 4x^3 + x^4)$
 $+ (1 - 4x + 6x^2 - 4x^3 + x^4) = 82$
 $2 + 12x^2 + 2x^4 = 82$
 $x^4 + 6x^2 - 40 = 0$
 $(x^2 + 10)(x^2 - 4) = 0$
 $x^2 = -10$ [no real solutions] or $x^2 = 4$
 $x = \pm 2$
- 17** a $a \times (1.5)^2 = 18$
 $a = 18 \div 2.25 = 8$
 b $S_6 = \frac{8[(1.5)^6 - 1]}{1.5 - 1} = 166.25$
 c $8 \times (1.5)^{k-1} > 8000$
 $(k-1) \lg 1.5 > \lg 1000$
 $k > \frac{\lg 1000}{\lg 1.5} + 1$
 $k > 18.04 \therefore$ smallest $k = 19$
- 18** $(1 + \frac{ax}{2})^{10} + (1 + bx)^{10}$
 $= 1 + 10(\frac{ax}{2}) + \frac{10 \times 9}{2}(\frac{ax}{2})^2 + \dots$
 $+ 1 + 10(bx) + \frac{10 \times 9}{2}(bx)^2 + \dots$
 $= 2 + (5a + 10b)x + (\frac{45}{4}a^2 + 45b^2)x^2 + \dots$
 $\therefore 5a + 10b = 0 \Rightarrow a = -2b$
 and $\frac{45}{4}a^2 + 45b^2 = 90 \Rightarrow a^2 + 4b^2 = 8$
 sub. $(-2b)^2 + 4b^2 = 8$
 $b^2 = 1$
 $a < b \therefore b = 1, a = -2$