

C2 TRIGONOMETRY

Answers - Worksheet E

- 1**
- a** $x = 30, 180 - 30$
 $x = 30^\circ, 150^\circ$
- b** $x = 60, 180 + 60$
 $x = 60^\circ, 240^\circ$
- c** $x = 90^\circ, 270^\circ$
- d** $x = 270^\circ$
- e** $x = 30, 360 - 30$
 $x = 30^\circ, 330^\circ$
- f** $x = 45, 180 - 45$
 $x = 45^\circ, 135^\circ$
- g** $x = 180 - 45, 360 - 45$
 $x = 135^\circ, 315^\circ$
- h** $x = 180 - 60, 180 + 60$
 $x = 120^\circ, 240^\circ$
- i** $x = 180 + 60, 360 - 60$
 $x = 240^\circ, 300^\circ$
- j** $x = 30, 180 + 30$
 $x = 30^\circ, 210^\circ$
- k** $x = 180 - 45, 180 + 45$
 $x = 135^\circ, 225^\circ$
- l** $x = 180 - 60, 360 - 60$
 $x = 120^\circ, 300^\circ$
- 2**
- a** $\theta = 66.4, 360 - 66.4$
 $\theta = 66.4^\circ, 293.6^\circ$
- b** $\theta = 15.7, 180 - 15.7$
 $\theta = 15.7^\circ, 164.3^\circ$
- c** $\theta = 58.0, 180 + 58.0$
 $\theta = 58.0^\circ, 238.0^\circ$
- d** $\theta = 54.4, 180 - 54.4$
 $\theta = 54.4^\circ, 125.6^\circ$
- e** $\theta = 5.7, 180 + 5.7$
 $\theta = 5.7^\circ, 185.7^\circ$
- f** $\theta = 79.3, 360 - 79.3$
 $\theta = 79.3^\circ, 280.7^\circ$
- g** $\theta = 180 + 36.9,$
 $360 - 36.9$
 $\theta = 216.9^\circ, 323.1^\circ$
- h** $\theta = 180 - 35.0,$
 $360 - 35.0$
 $\theta = 145.0^\circ, 325.0^\circ$
- i** $\theta = 180 - 67.0,$
 $180 + 67.0$
 $\theta = 113.0^\circ, 247.0^\circ$
- j** $\theta = 180 - 73.6,$
 $360 - 73.6$
 $\theta = 106.4^\circ, 286.4^\circ$
- k** $\theta = 180 - 50.5,$
 $180 + 50.5$
 $\theta = 129.5^\circ, 230.5^\circ$
- l** $\theta = 180 + 11.7,$
 $360 - 11.7$
 $\theta = 191.7^\circ, 348.3^\circ$
- 3**
- a** $x - 60 = 30, 180 - 30$
 $= 30, 150$
 $x = 90, 210$
- b** $x + 30 = 45, 180 + 45$
 $= 45, 225$
 $x = 15, 195$
- c** $x - 45 = 78.5, 360 - 78.5$
 $= 78.5, 281.5$
 $x = 123.5, 326.5$
- d** $x + 30 = 38.0, 180 + 38.0$
 $= 38.0, 218.0$
 $x = 8.0, 188.0$
- e** $x + 45 = 180 - 60, 180 + 60$
 $= 120, 240$
 $x = 75, 195$
- f** $x - 60 = 180 + 62.9, 360 - 62.9$
 $= 242.9, 297.1$
 $x = 302.9, 357.1$
- g** $x + 45 = 360 - 25.8,$
 $360 + 25.8$
 $= 334.2, 385.8$
 $x = 289.2, 340.8$
- h** $x + 30 = 180 - 8.0,$
 $360 + 8.0$
 $= 172.0, 368.0$
 $x = 142.0, 338.0$
- i** $x - 60 = -53.1, 53.1$
 $x = 6.9, 113.1$
- j** $x - 30 = -17.5, 180 + 17.5$
 $= -17.5, 197.5$
 $x = 12.5, 227.5$
- k** $x - 60 = -51.6, 180 - 51.6$
 $= -51.6, 128.4$
 $x = 8.4, 188.4$
- l** $2x = 30, 180 - 30,$
 $360 + 30, 540 - 30$
 $= 30, 150, 390, 510$
 $x = 15, 75, 195, 255$
- m** $2x = 50.208,$
 $360 - 50.208,$
 $360 + 50.208,$
 $720 - 50.208$
 $= 50.208, 309.792,$
 $410.208, 669.792$
 $x = 25.1, 154.9, 205.1, 334.9$
- n** $2x = 180 + 10.370,$
 $360 - 10.370,$
 $540 + 10.370,$
 $720 - 10.370$
 $= 190.370, 349.630,$
 $550.370, 709.630$
 $x = 95.2, 174.8, 275.2, 354.8$
- o** $2x = 180 - 69.950,$
 $360 - 69.950,$
 $540 - 69.950,$
 $720 - 69.950$
 $= 110.050, 290.050,$
 $470.050, 650.050$
 $x = 55.0, 145.0, 235.0, 325.0$
- p** $\frac{1}{2}x = 44.668, 180 - 44.668$
 $= 44.668, 135.332$
 $x = 89.3, 270.7$
- q** $3x = 30.583, 180 + 30.583,$
 $360 + 30.583,$
 $540 + 30.583,$
 $720 + 30.583,$
 $900 + 30.583$
 $= 30.583, 210.583,$
 $390.583, 570.583,$
 $750.583, 930.583$
 $x = 10.2, 70.2, 130.2$
 $190.2, 250.2, 310.2$
- r** $2x = 180 - 65.481,$
 $180 + 65.481,$
 $540 - 65.481,$
 $540 + 65.481$
 $= 114.519, 245.481,$
 $474.519, 605.481$
 $x = 57.3, 122.7, 237.3, 302.7$

4 a $x = 0, \pi, 2\pi$

b $x = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$
 $x = \frac{\pi}{3}, \frac{5\pi}{3}$

c $x = \frac{\pi}{4}, \pi + \frac{\pi}{4}$
 $x = \frac{\pi}{4}, \frac{5\pi}{4}$

d $x = \pi$

e $x = \pi - \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$
 $x = \frac{5\pi}{6}, \frac{11\pi}{6}$

f $x = \pi + \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$
 $x = \frac{5\pi}{4}, \frac{7\pi}{4}$

g $x + \frac{\pi}{6} = \frac{\pi}{3}, \pi + \frac{\pi}{3}$
 $= \frac{\pi}{3}, \frac{4\pi}{3}$
 $x = \frac{\pi}{6}, \frac{7\pi}{6}$

h $x - \frac{\pi}{4} = \frac{\pi}{6}, \pi - \frac{\pi}{6}$
 $= \frac{\pi}{6}, \frac{5\pi}{6}$
 $x = \frac{5\pi}{12}, \frac{13\pi}{12}$

i $x + \frac{\pi}{3} = \pi - \frac{\pi}{6}, \pi + \frac{\pi}{6}$
 $= \frac{5\pi}{6}, \frac{7\pi}{6}$
 $x = \frac{\pi}{2}, \frac{5\pi}{6}$

j $x + \frac{\pi}{3} = \pi - \frac{\pi}{4}, 2\pi + \frac{\pi}{4}$
 $= \frac{3\pi}{4}, \frac{9\pi}{4}$
 $x = \frac{5\pi}{12}, \frac{23\pi}{12}$

k $2x = \pi - \frac{\pi}{4}, \pi + \frac{\pi}{4},$
 $3\pi - \frac{\pi}{4}, 3\pi + \frac{\pi}{4}$
 $= \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{11\pi}{4}, \frac{13\pi}{4}$
 $x = \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}$

l $3x = \frac{\pi}{6}, \pi + \frac{\pi}{6}, 2\pi + \frac{\pi}{6},$
 $3\pi + \frac{\pi}{6}, 4\pi + \frac{\pi}{6}, 5\pi + \frac{\pi}{6}$
 $= \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}, \frac{19\pi}{6}, \frac{25\pi}{6}, \frac{31\pi}{6}$
 $x = \frac{\pi}{18}, \frac{7\pi}{18}, \frac{13\pi}{18}, \frac{19\pi}{18}, \frac{25\pi}{18}, \frac{31\pi}{18}$

5 a $\theta = -90^\circ, 90^\circ$

b $\tan 2\theta = -1$
 $2\theta = 180 - 45, 360 - 45$
 $-45, -45 - 180$
 $= -225, -45, 135, 315$
 $\theta = -112.5^\circ, -22.5^\circ,$
 $67.5^\circ, 157.5^\circ$

c $\theta + 60 = 16.9, 180 - 16.9$
 $= 16.9, 163.1$
 $\theta = -43.1^\circ, 103.1^\circ$

d $\tan(\theta - 15) = 1.85$
 $\theta - 15 = 61.6, 61.6 - 180$
 $= -118.4, 61.6$
 $\theta = -103.4^\circ, 76.6^\circ$

e $\sin 2\theta = 0.3$
 $2\theta = 17.458, 180 - 17.458,$
 $17.458 - 360,$
 $-17.458 - 180$
 $= -342.542, -197.458,$
 $17.458, 162.542$
 $\theta = -171.3^\circ, -98.7^\circ$
 $8.7^\circ, 81.3^\circ$

f $\cos 3\theta = 0.5$
 $3\theta = 60, 360 - 60, 360 + 60,$
 $-60, 60 - 360, -60 - 360$
 $= -420, -300, -60,$
 $60, 300, 420$
 $\theta = -140^\circ, -100^\circ, -20^\circ$
 $20^\circ, 100^\circ, 140^\circ$

g $\sin(\theta + 110) = -1$
 $\theta + 110 = 270$
 $\theta = 160^\circ$

h $\cos(\theta - 27) = 0.6$
 $\theta - 27 = 53.1, -53.1$
 $\theta = -26.1^\circ, 80.1^\circ$

i $\tan \theta = \frac{7}{3}$
 $\theta = 66.8, 66.8 - 180$
 $\theta = -113.2^\circ, 66.8^\circ$

j $\cos 2\theta = -0.375$
 $2\theta = 180 - 67.976,$
 $180 + 67.976,$
 $67.976 - 180,$
 $-67.976 - 180$
 $= -247.976, -112.024,$
 $112.024, 247.976$
 $\theta = -124.0^\circ, -56.0^\circ,$
 $56.0^\circ, 124.0^\circ$

k $\tan(\theta + 92) = -\frac{1}{3}$
 $\theta + 92 = 180 - 18.4, -18.4$
 $= -18.4, 161.6$
 $\theta = -110.4^\circ, 69.6^\circ$

l $\sin \frac{1}{3}\theta = 0.25$
 $\frac{1}{3}\theta = 14.478$
 $\theta = 43.4^\circ$

- 6**
- a** $2x + 30 = 45, 180 + 45$
 $= 45, 225$
 $2x = 15, 195$
 $x = 7.5^\circ, 97.5^\circ$
- b** $2x - 15 = 0, 180$
 $2x = 15, 195$
 $x = 7.5^\circ, 97.5^\circ$
- c** $2x + 70 = 360 - 60, 360 + 60$
 $= 300, 420$
 $2x = 230, 350$
 $x = 115^\circ, 175^\circ$
- d** $2x + 210 = 360 + 15.070,$
 $540 - 15.070$
 $= 375.070, 524.930$
 $2x = 165.070, 314.930$
 $x = 82.5^\circ, 157.5^\circ$
- e** $2x - 38 = 180 - 50.208,$
 $180 + 50.208$
 $= 129.792, 230.208$
 $2x = 167.792, 268.208$
 $x = 83.9^\circ, 134.1^\circ$
- f** $2x - 56 = 180 - 17.745,$
 -17.745
 $= -17.745, 162.256$
 $2x = 38.256, 218.256$
 $x = 19.1^\circ, 109.1^\circ$
- g** $3x - 24 = 42.862,$
 $360 - 42.862,$
 $360 + 42.862$
 $= 42.862, 317.138,$
 402.862
 $3x = 66.862, 341.138,$
 426.862
 $x = 22.3^\circ, 113.7^\circ, 142.3^\circ$
- h** $3x + 60 = 180 - 62.241,$
 $360 - 62.241,$
 $540 - 62.241$
 $= 117.759, 297.759,$
 477.759
 $3x = 57.759, 237.759,$
 417.759
 $x = 19.3^\circ, 79.3^\circ, 139.3^\circ$
- i** $\frac{1}{2}x + 18 = 34.890$
 $\frac{1}{2}x = 16.890$
 $x = 33.8^\circ$
- 7**
- a** $x = 0.48, \pi + 0.4795$
 $x = 0.48^\circ, 3.62^\circ$
- b** $2x = 1.2503, 2\pi - 1.2503,$
 $2\pi + 1.2503, 4\pi - 1.25032$
 $= 1.2503, 5.0328,$
 $7.5335, 11.3160$
 $x = 0.63^\circ, 2.52^\circ, 3.77^\circ, 5.66^\circ$
- c** $x + \frac{\pi}{4} = \pi - 0.7754,$
 $2\pi + 0.7754$
 $= 2.3662, 7.0586$
 $x = 1.58^\circ, 6.27^\circ$
- d** $\cos x = -\frac{1}{3}$
 $x = \pi - 1.2310, \pi + 1.2310$
 $= 1.91^\circ, 4.37^\circ$
- e** $\frac{1}{2}x = 0.0901, \pi - 0.0901$
 $= 0.0901, 3.0515$
 $x = 0.18^\circ, 6.10^\circ$
- f** $2x = \pi - 0.2213, 2\pi - 0.2213$
 $3\pi - 0.2213, 4\pi - 0.2213$
 $= 2.9203, 6.0619,$
 $9.2035, 12.3451$
 $x = 1.46^\circ, 3.03^\circ, 4.60^\circ, 6.17^\circ$
- g** $\sin(x - \frac{\pi}{3}) = 0.75$
 $x - \frac{\pi}{3} = 0.8481, \pi - 0.8481$
 $= 0.8481, 2.2935$
 $x = 1.90^\circ, 3.34^\circ$
- h** $2x + \frac{\pi}{6} = 1.1071, \pi + 1.1071,$
 $2\pi + 1.1071, 3\pi + 1.1071$
 $= 1.1071, 4.2487,$
 $7.3903, 10.5319$
 $2x = 0.5835, 3.7251,$
 $6.8667, 10.0083$
 $x = 0.29^\circ, 1.86^\circ, 3.43^\circ, 5.00^\circ$
- i** $3x = \pi - 0.6266, \pi + 0.6266,$
 $3\pi - 0.6266, 3\pi + 0.6266,$
 $5\pi - 0.6266, 5\pi + 0.6266$
 $= 2.5149, 3.7682, 8.7981,$
 $10.0514, 15.0813, 16.3346$
 $x = 0.84^\circ, 1.26^\circ, 2.93^\circ,$
 $3.35^\circ, 5.03^\circ, 5.44^\circ$
- j** $\tan x = -\frac{5}{3}$
 $x = \pi - 1.0304, 2\pi - 1.0304$
 $x = 2.11^\circ, 5.25^\circ$
- k** $2x - \frac{\pi}{2} = \pi - 1.2239, \pi + 1.2239,$
 $3\pi - 1.2239, 3\pi + 1.2239$
 $= 1.9177, 4.3655,$
 $8.2009, 10.6487$
 $2x = 3.4885, 5.9363,$
 $9.7717, 12.2195$
 $x = 1.74^\circ, 2.97^\circ, 4.89^\circ, 6.11^\circ$
- l** $\sin 2x = -\frac{1}{6}$
 $2x = \pi + 0.1674, 2\pi - 0.1674,$
 $3\pi + 0.1674, 4\pi - 0.1674$
 $= 3.3090, 6.1157,$
 $9.5922, 12.3989$
 $x = 1.65^\circ, 3.06^\circ, 4.80^\circ, 6.20^\circ$

8 a $(2y - 1)(y - 1) = 0$

$$y = \frac{1}{2}, 1$$

b $\sin x = \frac{1}{2}$ or 1

$$x = 30, 180 - 30 \text{ or } 90$$

$$x = 30^\circ, 90^\circ, 150^\circ$$

9 a $\sin \theta = \pm \frac{\sqrt{3}}{2}$

$$\theta = 60, 180 - 60 \text{ or } 180 + 60, 360 - 60$$

$$\theta = 60, 120, 240, 300$$

c $\cos \theta (2 \cos \theta + 1) = 0$

$$\cos \theta = 0 \text{ or } -0.5$$

$$\theta = 90, 270 \text{ or } 180 - 60, 180 + 60$$

$$\theta = 90, 120, 240, 270$$

e $\sin \theta (4 - \tan \theta) = 0$

$$\sin \theta = 0 \text{ or } \tan \theta = 4$$

$$\theta = 0, 180, 360 \text{ or } 76.0, 180 + 76.0$$

$$\theta = 0, 76.0, 180, 256.0, 360$$

g $(\tan \theta - 1)(\tan \theta - 2) = 0$

$$\tan \theta = 1 \text{ or } 2$$

$$\theta = 45, 180 + 45 \text{ or } 63.4, 180 + 63.4$$

$$\theta = 45, 63.4, 225, 243.4$$

i $\tan^2 \theta - \tan \theta - 6 = 0$

$$(\tan \theta + 2)(\tan \theta - 3) = 0$$

$$\tan \theta = -2 \text{ or } 3$$

$$\theta = 180 - 63.4, 360 - 63.4 \text{ or } 71.6, 180 + 71.6$$

$$\theta = 71.6, 116.6, 251.6, 296.6$$

k $4 \sin^2 \theta - 8 \sin \theta + 3 = 0$

$$(2 \sin \theta - 1)(2 \sin \theta - 3) = 0$$

$$\sin \theta = 0.5 \text{ or } 1.5 \text{ [no solutions]}$$

$$\theta = 30, 180 - 30$$

$$\theta = 30, 150$$

m $\tan \theta = \frac{-3 \pm \sqrt{9+4}}{2}$

$$\tan \theta = \frac{1}{2}(-3 \pm \sqrt{13})$$

$$\theta = 180 - 73.2, 360 - 73.2 \text{ or } 16.8, 180 + 16.8$$

$$\theta = 16.8, 106.8, 196.8, 286.8$$

b $\tan \theta = \pm 1$

$$\theta = 45, 180 + 45 \text{ or } 180 - 45, 360 - 45$$

$$\theta = 45, 135, 225, 315$$

d $\sin \theta = 0$ or $\cos \theta = 0.25$

$$\theta = 0, 180, 360 \text{ or } 75.5, 360 - 75.5$$

$$\theta = 0, 75.5, 180, 284.5, 360$$

f $\cos \theta = -1$ or 0.5

$$\theta = 180 \text{ or } 60, 360 - 60$$

$$\theta = 60, 180, 300$$

h $(3 \sin \theta - 1)(\sin \theta - 2) = 0$

$$\sin \theta = \frac{1}{3} \text{ or } 2 \text{ [no solutions]}$$

$$\theta = 19.5, 180 - 19.5$$

$$\theta = 19.5, 160.5$$

j $(3 \cos \theta - 2)(2 \cos \theta + 1) = 0$

$$\cos \theta = -0.5 \text{ or } \frac{2}{3}$$

$$\theta = 180 - 60, 180 + 60 \text{ or } 48.2, 360 - 48.2$$

$$\theta = 48.2, 120, 240, 311.8$$

l $\cos \theta = \frac{-2 \pm \sqrt{4+4}}{2}$

$$\cos \theta = -1 + \sqrt{2} \text{ or } -1 - \sqrt{2} \text{ [no solutions]}$$

$$\theta = 65.5, 360 - 65.5$$

$$\theta = 65.5, 294.5$$

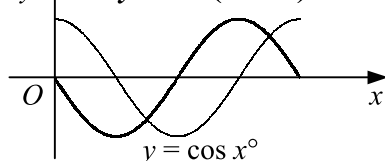
n $3 \sin^2 \theta + \sin \theta - 1 = 0$

$$\sin \theta = \frac{-1 \pm \sqrt{1+12}}{6} = \frac{1}{6}(-1 \pm \sqrt{13})$$

$$\theta = 180 + 50.1, 360 - 50.1 \text{ or } 25.7, 180 - 25.7$$

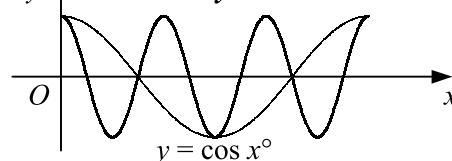
$$\theta = 25.7, 154.3, 230.1, 309.9$$

10 a, b $y = \cos(x + 90^\circ)$



c $x = 135, 315$

11 a $y = \cos 3x^\circ$



b $x = 0, 90, 180, 270, 360$

c $x = 0, 45, 90, 135, 180$

C2 TRIGONOMETRY

Answers - Worksheet F

- 1 a** $4 \sin x = -\cos x$
 $\frac{\sin x}{\cos x} = -\frac{1}{4}$
 $\tan x = -\frac{1}{4}$
- b** $x = 180 - 14.0, 360 - 14.0$
 $x = 166.0^\circ, 346.0^\circ$
- 2 a** LHS = $5 \sin^2 x + 5 \sin x + 4(1 - \sin^2 x)$
 $= \sin^2 x + 5 \sin x + 4$
 $= \text{RHS}$
- b** $(\sin x + 4)(\sin x + 1) = 0$
 $\sin x = -1$ or -4 [no solutions]
 $x = 270^\circ$
- 3 a** $2 \sin x = \cos x$
 $\tan x = 0.5$
 $x = 26.6, 180 + 26.6$
 $x = 26.6^\circ, 206.6^\circ$
- c** $1 - \sin^2 x + 3 \sin x - 3 = 0$
 $\sin^2 x - 3 \sin x + 2 = 0$
 $(\sin x - 1)(\sin x - 2) = 0$
 $\sin x = 1$ or 2 [no solutions]
 $x = 90^\circ$
- d** $3 \cos^2 x - (1 - \cos^2 x) = 2$
 $4 \cos^2 x = 3$
 $\cos x = \pm \frac{\sqrt{3}}{2}$
 $x = 30, 360 - 30$ or $180 - 30, 180 + 30$
 $x = 30^\circ, 150^\circ, 210^\circ, 330^\circ$
- e** $2(1 - \cos^2 x) + 3 \cos x = 3$
 $2 \cos^2 x - 3 \cos x + 1 = 0$
 $(2 \cos x - 1)(\cos x - 1) = 0$
 $\cos x = 0.5$ or 1
 $x = 60, 360 - 60$ or $0, 360$
 $x = 0, 60^\circ, 300^\circ, 360^\circ$
- f** $3(1 - \sin^2 x) = 5(1 - \sin x)$
 $3 \sin^2 x - 5 \sin x + 2 = 0$
 $(3 \sin x - 2)(\sin x - 1) = 0$
 $\sin x = \frac{2}{3}$ or 1
 $x = 41.8, 180 - 41.8$ or 90
 $x = 41.8^\circ, 90^\circ, 138.2^\circ$
- g** $3 \sin^2 x = 8 \cos x$
 $3(1 - \cos^2 x) = 8 \cos x$
 $3 \cos^2 x + 8 \cos x - 3 = 0$
 $(3 \cos x - 1)(\cos x + 3) = 0$
 $\cos x = \frac{1}{3}$ or -3 [no solutions]
 $x = 70.5, 360 - 70.5$
 $x = 70.5^\circ, 289.5^\circ$
- h** $\cos^2 x = 3 \sin x$
 $1 - \sin^2 x = 3 \sin x$
 $\sin^2 x + 3 \sin x - 1 = 0$
 $\sin x = \frac{-3 \pm \sqrt{9+4}}{2}$
 $\sin x = \frac{1}{2}(-3 + \sqrt{13})$ or $\frac{1}{2}(-3 - \sqrt{13})$ [no sols]
 $x = 17.6, 180 - 17.6$
 $x = 17.6^\circ, 162.4^\circ$
- i** $3(1 - \cos^2 x) - 5 \cos x + 2 \cos^2 x = 0$
 $\cos^2 x + 5 \cos x - 3 = 0$
 $\cos x = \frac{-5 \pm \sqrt{25+12}}{2}$
 $\cos x = \frac{1}{2}(-5 + \sqrt{37})$ or $\frac{1}{2}(-5 - \sqrt{37})$ [no sols]
 $x = 57.2, 360 - 57.2$
 $x = 57.2^\circ, 302.8^\circ$
- j** $2 \sin^2 x + 7 \sin x - 2(1 - \sin^2 x) = 0$
 $4 \sin^2 x + 7 \sin x - 2 = 0$
 $(4 \sin x - 1)(\sin x + 2) = 0$
 $\sin x = 0.25$ or -2 [no solutions]
 $x = 14.5, 180 - 14.5$
 $x = 14.5^\circ, 165.5^\circ$
- k** $3 \sin x = 2 \tan x$
 $3 \sin x \cos x = 2 \sin x$
 $\sin x (3 \cos x - 2) = 0$
 $\sin x = 0$ or $\cos x = \frac{2}{3}$
 $x = 0, 180, 360$ or $48.2, 360 - 48.2$
 $x = 0, 48.2^\circ, 180^\circ, 311.8^\circ, 360^\circ$
- l** $(1 - \cos^2 x) - 9 \cos x - \cos^2 x = 5$
 $2 \cos^2 x + 9 \cos x + 4 = 0$
 $(2 \cos x + 1)(\cos x + 4) = 0$
 $\cos x = -0.5$ or -4 [no solutions]
 $x = 180 - 60, 180 + 60$
 $x = 120^\circ, 240^\circ$

- 4 a** $\cos \theta = \pm 0.5$
 $\theta = \frac{\pi}{3}, -\frac{\pi}{3}$ or $\pi - \frac{\pi}{3}, -\pi + \frac{\pi}{3}$
 $\theta = -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$
- b** $(2 \sin \theta + 1)^2 = 0$
 $\sin \theta = -0.5$
 $\theta = -\frac{\pi}{6}, -\pi + \frac{\pi}{6}$
 $\theta = -\frac{5\pi}{6}, -\frac{\pi}{6}$
- c** $(\cos \theta + 3)(\cos \theta - 1) = 0$
 $\cos \theta = 1$ or -3 [no solutions]
 $\theta = 0$
- d** $3 \sin^2 \theta - (1 - \sin^2 \theta) = 0$
 $4 \sin^2 \theta = 1$
 $\sin \theta = \pm 0.5$
 $\theta = \frac{\pi}{6}, \pi - \frac{\pi}{6}$ or $-\frac{\pi}{6}, -\pi + \frac{\pi}{6}$
 $\theta = -\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$
- e** $4 \sin^2 \theta - 5 \sin \theta + 2(1 - \sin^2 \theta) = 0$
 $2 \sin^2 \theta - 5 \sin \theta + 2 = 0$
 $(2 \sin \theta - 1)(\sin \theta - 2) = 0$
 $\sin \theta = 0.5$ or 2 [no solutions]
 $\theta = \frac{\pi}{6}, \pi - \frac{\pi}{6}$
 $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$
- f** $(1 - \cos^2 \theta) - 3 \cos \theta - \cos^2 \theta = 2$
 $2 \cos^2 \theta + 3 \cos \theta + 1 = 0$
 $(2 \cos \theta + 1)(\cos \theta + 1) = 0$
 $\cos \theta = -0.5$ or -1
 $\theta = \pi - \frac{\pi}{3}, -\pi + \frac{\pi}{3}$ or $-\pi, \pi$
 $\theta = -\pi, -\frac{2\pi}{3}, \frac{2\pi}{3}, \pi$
- 5 a** LHS = $\sin^2 x + 2 \sin x \cos x + \cos^2 x$
 $= (\sin^2 x + \cos^2 x) + 2 \sin x \cos x$
 $= 1 + 2 \sin x \cos x$
 $= \text{RHS}$
- b** LHS = $\frac{1 - \cos^2 x}{\cos x}$
 $= \frac{\sin^2 x}{\cos x}$
 $= \sin x \times \frac{\sin x}{\cos x}$
 $= \sin x \tan x$
 $= \text{RHS}$
- c** LHS = $\frac{1 - \sin^2 x}{1 - \sin x}$
 $= \frac{(1 + \sin x)(1 - \sin x)}{1 - \sin x}$
 $= 1 + \sin x$
 $= \text{RHS}$
- d** LHS = $\frac{(1 + \sin x)(1 - \sin x)}{\cos x(1 - \sin x)}$
 $= \frac{1 - \sin^2 x}{\cos x(1 - \sin x)}$
 $= \frac{\cos^2 x}{\cos x(1 - \sin x)}$
 $= \frac{\cos x}{1 - \sin x}$
 $= \text{RHS}$
- 6 a** LHS = $\cos^2 x - 2 \cos x \tan x + \tan^2 x$
 $+ \sin^2 x + 2 \sin x + 1$
 $= \cos^2 x - 2 \sin x + \tan^2 x$
 $+ \sin^2 x + 2 \sin x + 1$
 $= (\cos^2 x + \sin^2 x) + \tan^2 x + 1$
 $= 2 + \tan^2 x = \text{RHS}$
- b** $2 + \tan^2 x = 3$
 $\tan^2 x = 1$
 $\tan x = \pm 1$
 $x = \frac{\pi}{4}, \pi + \frac{\pi}{4}$ or $\pi - \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$
 $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$
- 7 a** $f(x) = (1 - \sin^2 x) + 2 \sin x$
 $= 2 - (\sin^2 x - 2 \sin x + 1)$
 $= 2 - (\sin x - 1)^2$
- b** max. value of $f(x) = 2$
occurs when $\sin x = 1 \therefore x = \frac{\pi}{2}$