

C2 TRIGONOMETRY

Answers - Worksheet F

- 1 a** $4 \sin x = -\cos x$
 $\frac{\sin x}{\cos x} = -\frac{1}{4}$
 $\tan x = -\frac{1}{4}$
- b** $x = 180 - 14.0, 360 - 14.0$
 $x = 166.0^\circ, 346.0^\circ$
- 2 a** LHS = $5 \sin^2 x + 5 \sin x + 4(1 - \sin^2 x)$
 $= \sin^2 x + 5 \sin x + 4$
 $= \text{RHS}$
- b** $(\sin x + 4)(\sin x + 1) = 0$
 $\sin x = -1$ or -4 [no solutions]
 $x = 270^\circ$
- 3 a** $2 \sin x = \cos x$
 $\tan x = 0.5$
 $x = 26.6, 180 + 26.6$
 $x = 26.6^\circ, 206.6^\circ$
- c** $1 - \sin^2 x + 3 \sin x - 3 = 0$
 $\sin^2 x - 3 \sin x + 2 = 0$
 $(\sin x - 1)(\sin x - 2) = 0$
 $\sin x = 1$ or 2 [no solutions]
 $x = 90^\circ$
- d** $3 \cos^2 x - (1 - \cos^2 x) = 2$
 $4 \cos^2 x = 3$
 $\cos x = \pm \frac{\sqrt{3}}{2}$
 $x = 30, 360 - 30$ or $180 - 30, 180 + 30$
 $x = 30^\circ, 150^\circ, 210^\circ, 330^\circ$
- e** $2(1 - \cos^2 x) + 3 \cos x = 3$
 $2 \cos^2 x - 3 \cos x + 1 = 0$
 $(2 \cos x - 1)(\cos x - 1) = 0$
 $\cos x = 0.5$ or 1
 $x = 60, 360 - 60$ or $0, 360$
 $x = 0, 60^\circ, 300^\circ, 360^\circ$
- f** $3(1 - \sin^2 x) = 5(1 - \sin x)$
 $3 \sin^2 x - 5 \sin x + 2 = 0$
 $(3 \sin x - 2)(\sin x - 1) = 0$
 $\sin x = \frac{2}{3}$ or 1
 $x = 41.8, 180 - 41.8$ or 90
 $x = 41.8^\circ, 90^\circ, 138.2^\circ$
- g** $3 \sin^2 x = 8 \cos x$
 $3(1 - \cos^2 x) = 8 \cos x$
 $3 \cos^2 x + 8 \cos x - 3 = 0$
 $(3 \cos x - 1)(\cos x + 3) = 0$
 $\cos x = \frac{1}{3}$ or -3 [no solutions]
 $x = 70.5, 360 - 70.5$
 $x = 70.5^\circ, 289.5^\circ$
- h** $\cos^2 x = 3 \sin x$
 $1 - \sin^2 x = 3 \sin x$
 $\sin^2 x + 3 \sin x - 1 = 0$
 $\sin x = \frac{-3 \pm \sqrt{9+4}}{2}$
 $\sin x = \frac{1}{2}(-3 + \sqrt{13})$ or $\frac{1}{2}(-3 - \sqrt{13})$ [no sols]
 $x = 17.6, 180 - 17.6$
 $x = 17.6^\circ, 162.4^\circ$
- i** $3(1 - \cos^2 x) - 5 \cos x + 2 \cos^2 x = 0$
 $\cos^2 x + 5 \cos x - 3 = 0$
 $\cos x = \frac{-5 \pm \sqrt{25+12}}{2}$
 $\cos x = \frac{1}{2}(-5 + \sqrt{37})$ or $\frac{1}{2}(-5 - \sqrt{37})$ [no sols]
 $x = 57.2, 360 - 57.2$
 $x = 57.2^\circ, 302.8^\circ$
- j** $2 \sin^2 x + 7 \sin x - 2(1 - \sin^2 x) = 0$
 $4 \sin^2 x + 7 \sin x - 2 = 0$
 $(4 \sin x - 1)(\sin x + 2) = 0$
 $\sin x = 0.25$ or -2 [no solutions]
 $x = 14.5, 180 - 14.5$
 $x = 14.5^\circ, 165.5^\circ$
- k** $3 \sin x = 2 \tan x$
 $3 \sin x \cos x = 2 \sin x$
 $\sin x (3 \cos x - 2) = 0$
 $\sin x = 0$ or $\cos x = \frac{2}{3}$
 $x = 0, 180, 360$ or $48.2, 360 - 48.2$
 $x = 0, 48.2^\circ, 180^\circ, 311.8^\circ, 360^\circ$
- l** $(1 - \cos^2 x) - 9 \cos x - \cos^2 x = 5$
 $2 \cos^2 x + 9 \cos x + 4 = 0$
 $(2 \cos x + 1)(\cos x + 4) = 0$
 $\cos x = -0.5$ or -4 [no solutions]
 $x = 180 - 60, 180 + 60$
 $x = 120^\circ, 240^\circ$

- 4 a** $\cos \theta = \pm 0.5$
 $\theta = \frac{\pi}{3}, -\frac{\pi}{3}$ or $\pi - \frac{\pi}{3}, -\pi + \frac{\pi}{3}$
 $\theta = -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$
- b** $(2 \sin \theta + 1)^2 = 0$
 $\sin \theta = -0.5$
 $\theta = -\frac{\pi}{6}, -\pi + \frac{\pi}{6}$
 $\theta = -\frac{5\pi}{6}, -\frac{\pi}{6}$
- c** $(\cos \theta + 3)(\cos \theta - 1) = 0$
 $\cos \theta = 1$ or -3 [no solutions]
 $\theta = 0$
- d** $3 \sin^2 \theta - (1 - \sin^2 \theta) = 0$
 $4 \sin^2 \theta = 1$
 $\sin \theta = \pm 0.5$
 $\theta = \frac{\pi}{6}, \pi - \frac{\pi}{6}$ or $-\frac{\pi}{6}, -\pi + \frac{\pi}{6}$
 $\theta = -\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$
- e** $4 \sin^2 \theta - 5 \sin \theta + 2(1 - \sin^2 \theta) = 0$
 $2 \sin^2 \theta - 5 \sin \theta + 2 = 0$
 $(2 \sin \theta - 1)(\sin \theta - 2) = 0$
 $\sin \theta = 0.5$ or 2 [no solutions]
 $\theta = \frac{\pi}{6}, \pi - \frac{\pi}{6}$
 $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$
- f** $(1 - \cos^2 \theta) - 3 \cos \theta - \cos^2 \theta = 2$
 $2 \cos^2 \theta + 3 \cos \theta + 1 = 0$
 $(2 \cos \theta + 1)(\cos \theta + 1) = 0$
 $\cos \theta = -0.5$ or -1
 $\theta = \pi - \frac{\pi}{3}, -\pi + \frac{\pi}{3}$ or $-\pi, \pi$
 $\theta = -\pi, -\frac{2\pi}{3}, \frac{2\pi}{3}, \pi$
- 5 a** LHS = $\sin^2 x + 2 \sin x \cos x + \cos^2 x$
 $= (\sin^2 x + \cos^2 x) + 2 \sin x \cos x$
 $= 1 + 2 \sin x \cos x$
 $= \text{RHS}$
- b** LHS = $\frac{1 - \cos^2 x}{\cos x}$
 $= \frac{\sin^2 x}{\cos x}$
 $= \sin x \times \frac{\sin x}{\cos x}$
 $= \sin x \tan x$
 $= \text{RHS}$
- c** LHS = $\frac{1 - \sin^2 x}{1 - \sin x}$
 $= \frac{(1 + \sin x)(1 - \sin x)}{1 - \sin x}$
 $= 1 + \sin x$
 $= \text{RHS}$
- d** LHS = $\frac{(1 + \sin x)(1 - \sin x)}{\cos x(1 - \sin x)}$
 $= \frac{1 - \sin^2 x}{\cos x(1 - \sin x)}$
 $= \frac{\cos^2 x}{\cos x(1 - \sin x)}$
 $= \frac{\cos x}{1 - \sin x}$
 $= \text{RHS}$
- 6 a** LHS = $\cos^2 x - 2 \cos x \tan x + \tan^2 x$
 $+ \sin^2 x + 2 \sin x + 1$
 $= \cos^2 x - 2 \sin x + \tan^2 x$
 $+ \sin^2 x + 2 \sin x + 1$
 $= (\cos^2 x + \sin^2 x) + \tan^2 x + 1$
 $= 2 + \tan^2 x = \text{RHS}$
- b** $2 + \tan^2 x = 3$
 $\tan^2 x = 1$
 $\tan x = \pm 1$
 $x = \frac{\pi}{4}, \pi + \frac{\pi}{4}$ or $\pi - \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$
 $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$
- 7 a** $f(x) = (1 - \sin^2 x) + 2 \sin x$
 $= 2 - (\sin^2 x - 2 \sin x + 1)$
 $= 2 - (\sin x - 1)^2$
- b** max. value of $f(x) = 2$
occurs when $\sin x = 1 \therefore x = \frac{\pi}{2}$