

C2 INTEGRATION

Answers - Worksheet A

- 1 a** $= [2x^2 - x]_1^3$
 $= (18 - 3) - (2 - 1)$
 $= 14$
- b** $= [x^3 + 2x]_0^1$
 $= (1 + 2) - (0)$
 $= 3$
- c** $= [\frac{1}{2}x^2 - \frac{1}{3}x^3]_0^3$
 $= (\frac{9}{2} - 9) - (0)$
 $= -\frac{9}{2}$
- d** $= \int_2^3 (9x^2 + 6x + 1) dx$
 $= [3x^3 + 3x^2 + x]_2^3$
 $= (81 + 27 + 3) - (24 + 12 + 2)$
 $= 73$
- e** $= [\frac{1}{3}x^3 - 4x^2 - 3x]_1^2$
 $= (\frac{8}{3} - 16 - 6) - (\frac{1}{3} - 4 - 3)$
 $= -12\frac{2}{3}$
- f** $= [8x - 2x^2 + x^3]_{-2}^4$
 $= (32 - 32 + 64) - (-16 - 8 - 8)$
 $= 96$
- g** $= [\frac{1}{4}x^4 - x^2 - 7x]_1^4$
 $= (64 - 16 - 28) - (\frac{1}{4} - 1 - 7)$
 $= 27\frac{3}{4}$
- h** $= [5x + \frac{1}{3}x^3 - x^4]_{-2}^{-1}$
 $= (-5 - \frac{1}{3} - 1) - (-10 - \frac{8}{3} - 16)$
 $= 22\frac{1}{3}$
- i** $= [\frac{1}{5}x^5 + 2x^3 - \frac{1}{2}x^2]_{-1}^2$
 $= (\frac{32}{5} + 16 - 2) - (-\frac{1}{5} - 2 - \frac{1}{2})$
 $= 23\frac{1}{10}$
- 2** $\int_1^4 (3x^2 + ax - 5) dx = [x^3 + \frac{1}{2}ax^2 - 5x]_1^4$
 $= (64 + 8a - 20) - (1 + \frac{1}{2}a - 5) = 48 + \frac{15}{2}a$
 $\therefore 48 + \frac{15}{2}a = 18$
 $a = -4$
- 3** $\int_{-1}^k (3x^2 - 12x + 9) dx = [x^3 - 6x^2 + 9x]_{-1}^k$
 $= (k^3 - 6k^2 + 9k) - (-1 - 6 - 9) = k^3 - 6k^2 + 9k + 16$
 $\therefore k^3 - 6k^2 + 9k + 16 = 16$
 $k(k^2 - 6k + 9) = 0$
 $k(k - 3)^2 = 0$
 $k \neq 0 \therefore k = 3$
- 4 a** $= \int_1^3 (2 - x^{-2}) dx$
 $= [2x + x^{-1}]_1^3$
 $= (6 + \frac{1}{3}) - (2 + 1)$
 $= \frac{10}{3}$
- b** $= \int_{-2}^{-1} (6x + 4x^{-3}) dx$
 $= [3x^2 - 2x^{-2}]_{-2}^{-1}$
 $= (3 - 2) - (12 - \frac{1}{2})$
 $= -10\frac{1}{2}$
- c** $= [2x^{\frac{3}{2}} - 4x]_1^4$
 $= (16 - 16) - (2 - 4)$
 $= 2$
- d** $= \int_{-1}^2 (2x^3 - \frac{1}{2}) dx$
 $= [\frac{1}{2}x^4 - \frac{1}{2}x]_{-1}^2$
 $= (8 - 1) - (\frac{1}{2} + \frac{1}{2})$
 $= 6$
- e** $= [\frac{1}{2}x^2 - \frac{3}{2}x^{\frac{3}{2}}]_1^8$
 $= (32 - 6) - (\frac{1}{2} - \frac{3}{2})$
 $= 27$
- f** $= \int_2^3 (\frac{1}{3}x^{-2} - 2x) dx$
 $= [-\frac{1}{3}x^{-1} - x^2]_2^3$
 $= (-\frac{1}{9} - 9) - (-\frac{1}{6} - 4)$
 $= -4\frac{17}{18}$
- 5** $= \int_1^3 (3x^2 - 6x + 7) dx$
 $= [x^3 - 3x^2 + 7x]_1^3$
 $= (27 - 27 + 21) - (1 - 3 + 7) = 16$

$$\begin{aligned} 6 \quad \mathbf{a} \quad &= \int_0^2 (x^2 + 2) \, dx \\ &= \left[\frac{1}{3}x^3 + 2x \right]_0^2 \\ &= \left(\frac{8}{3} + 4 \right) - 0 = 6\frac{2}{3} \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad &= \int_{-2}^1 (3x^2 + 8x + 6) \, dx \\ &= [x^3 + 4x^2 + 6x]_{-2}^1 \\ &= (1 + 4 + 6) - (-8 + 16 - 12) = 15 \end{aligned}$$

$$\begin{aligned} \mathbf{c} \quad &= \int_2^4 (9 + 2x - x^2) \, dx \\ &= \left[9x + x^2 - \frac{1}{3}x^3 \right]_2^4 \\ &= (36 + 16 - \frac{64}{3}) - (18 + 4 - \frac{8}{3}) = 11\frac{1}{3} \end{aligned}$$

$$\begin{aligned} \mathbf{d} \quad &= \int_{-1}^0 (x^3 - 4x + 1) \, dx \\ &= \left[\frac{1}{4}x^4 - 2x^2 + x \right]_{-1}^0 \\ &= 0 - \left(\frac{1}{4} - 2 - 1 \right) = \frac{11}{4} \end{aligned}$$

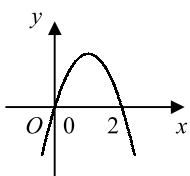
$$\begin{aligned} \mathbf{e} \quad &= \int_1^4 (2x + 3x^{\frac{1}{2}}) \, dx \\ &= \left[x^2 + 2x^{\frac{3}{2}} \right]_1^4 \\ &= (16 + 16) - (1 + 2) = 29 \end{aligned}$$

$$\begin{aligned} \mathbf{f} \quad &= \int_{-5}^{-1} (3 + 5x^{-2}) \, dx \\ &= [3x - 5x^{-1}]_{-5}^{-1} \\ &= (-3 + 5) - (-15 + 1) = 16 \end{aligned}$$

$$\begin{aligned} 7 \quad \mathbf{a} \quad y = 0 &\Rightarrow 4 - x^2 = 0 \\ &x^2 = 4 \\ &x = \pm 2 \\ \therefore &(-2, 0) \text{ and } (2, 0) \end{aligned}$$

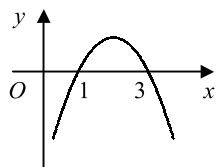
$$\begin{aligned} \mathbf{b} \quad &= \int_{-2}^2 (4 - x^2) \, dx \\ &= \left[4x - \frac{1}{3}x^3 \right]_{-2}^2 \\ &= \left(8 - \frac{8}{3} \right) - \left(-8 + \frac{8}{3} \right) \\ &= 10\frac{2}{3} \end{aligned}$$

$$\begin{aligned} 8 \quad \mathbf{a} \quad 6x - 3x^2 &= 0 \\ 3x(2 - x) &= 0 \\ x &= 0, 2 \end{aligned}$$



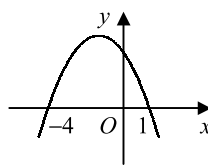
$$\begin{aligned} \text{area} &= \int_0^2 (6x - 3x^2) \, dx \\ &= \left[3x^2 - x^3 \right]_0^2 \\ &= (12 - 8) - 0 \\ &= 4 \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad -x^2 + 4x - 3 &= 0 \\ -(x - 1)(x - 3) &= 0 \\ x &= 1, 3 \end{aligned}$$



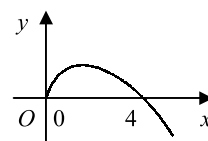
$$\begin{aligned} \text{area} &= \int_1^3 (-x^2 + 4x - 3) \, dx \\ &= \left[-\frac{1}{3}x^3 + 2x^2 - 3x \right]_1^3 \\ &= (-9 + 18 - 9) \\ &\quad - \left(-\frac{1}{3} + 2 - 3 \right) \\ &= \frac{4}{3} \end{aligned}$$

$$\begin{aligned} \mathbf{c} \quad 4 - 3x - x^2 &= 0 \\ -(x + 4)(x - 1) &= 0 \\ x &= -4, 1 \end{aligned}$$



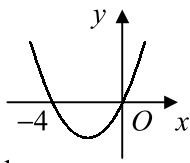
$$\begin{aligned} \text{area} &= \int_{-4}^1 (4 - 3x - x^2) \, dx \\ &= \left[4x - \frac{3}{2}x^2 - \frac{1}{3}x^3 \right]_{-4}^1 \\ &= \left(4 - \frac{3}{2} - \frac{1}{3} \right) \\ &\quad - \left(-16 - 24 + \frac{64}{3} \right) \\ &= 20\frac{5}{6} \end{aligned}$$

$$\begin{aligned} \mathbf{d} \quad 2x^{\frac{1}{2}} - x &= 0 \\ x^{\frac{1}{2}}(2 - x^{\frac{1}{2}}) &= 0 \\ x^{\frac{1}{2}} &= 0, 2 \\ x &= 0, 4 \end{aligned}$$



$$\begin{aligned} \text{area} &= \int_0^4 (2x^{\frac{1}{2}} - x) \, dx \\ &= \left[\frac{4}{3}x^{\frac{3}{2}} - \frac{1}{2}x^2 \right]_0^4 \\ &= \left(\frac{32}{3} - 8 \right) - 0 \\ &= \frac{8}{3} \end{aligned}$$

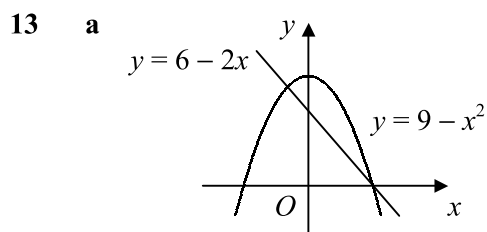
9 a $x^2 + 4x = 0$
 $x(x + 4) = 0$
 $x = -4, 0$



b $= \int_0^2 (x^2 + 4x) dx$
 $= [\frac{1}{3}x^3 + 2x^2]_0^2$
 $= (\frac{8}{3} + 8) - 0 = 10\frac{2}{3}$

11 a $x^3 - 5x^2 + 6x = 0$
 $x(x - 2)(x - 3) = 0$
 $x = 0, 2, 3$
 $\therefore (0, 0), (2, 0)$ and $(3, 0)$

b $\int_0^2 (x^3 - 5x^2 + 6x) dx$
 $= [\frac{1}{4}x^4 - \frac{5}{3}x^3 + 3x^2]_0^2$
 $= (4 - \frac{40}{3} + 12) - 0 = \frac{8}{3}$
 $\int_2^3 (x^3 - 5x^2 + 6x) dx$
 $= [\frac{1}{4}x^4 - \frac{5}{3}x^3 + 3x^2]_2^3$
 $= (\frac{81}{4} - 45 + 27) - \frac{8}{3} = -\frac{5}{12}$
 total area $= \frac{8}{3} + \frac{5}{12} = 3\frac{1}{12}$



$9 - x^2 = 6 - 2x$
 $x^2 - 2x - 3 = 0$
 $(x + 1)(x - 3) = 0$
 $x = -1, 3$

\therefore intersect at $(-1, 8)$ and $(3, 0)$
 area below curve

$= \int_{-1}^3 (9 - x^2) dx$
 $= [9x - \frac{1}{3}x^3]_{-1}^3$
 $= (27 - 9) - (-9 + \frac{1}{3})$
 $= 26\frac{2}{3}$

area below line
 $= \frac{1}{2} \times 4 \times 8 = 16$

area between line and curve
 $= 26\frac{2}{3} - 16 = 10\frac{2}{3}$

10 a $x^2 + 2x - 15 = 0$
 $(x + 5)(x - 3) = 0$
 $x = -5, 3$
 $\therefore (-5, 0)$ and $(3, 0)$

b $= [\frac{1}{3}x^3 + x^2 - 15x]_0^3$
 $= (9 + 9 - 45) - 0 = -27$

c 27

12 a $x^2 - 3x + 4 = x + 1$
 $x^2 - 4x + 3 = 0$
 $(x - 1)(x - 3) = 0$
 $x = 1, 3$

$\therefore (1, 2)$ and $(3, 4)$

b area below curve

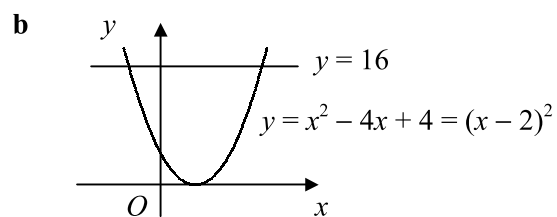
$= \int_1^3 (x^2 - 3x + 4) dx$

$= [\frac{1}{3}x^3 - \frac{3}{2}x^2 + 4x]_1^3$

$= (9 - \frac{27}{2} + 12) - (\frac{1}{3} - \frac{3}{2} + 4) = \frac{14}{3}$

area below line $= \frac{1}{2} \times 2 \times (2 + 4) = 6$

shaded area $= 6 - \frac{14}{3} = \frac{4}{3}$



$x^2 - 4x + 4 = 16$
 $x^2 - 4x - 12 = 0$
 $(x + 2)(x - 6) = 0$
 $x = -2, 6$

\therefore intersect at $(-2, 16)$ and $(6, 16)$
 area below curve

$= \int_{-2}^6 (x^2 - 4x + 4) dx$

$= [\frac{1}{3}x^3 - 2x^2 + 4x]_{-2}^6$

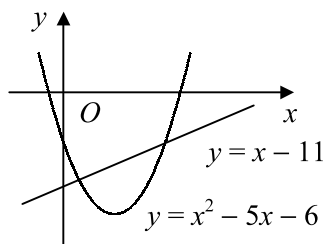
$= (72 - 72 + 24) - (-\frac{8}{3} - 8 - 8)$

$= 42\frac{2}{3}$

area below line
 $= 8 \times 16 = 128$

area between line and curve
 $= 128 - 42\frac{2}{3} = 85\frac{1}{3}$

c $y = x^2 - 5x - 6 \Rightarrow y = (x + 1)(x - 6)$



$$x^2 - 5x - 6 = x - 11$$

$$x^2 - 6x + 5 = 0$$

$$(x - 1)(x - 5) = 0$$

$$x = 1, 5$$

\therefore intersect at $(1, -10)$ and $(5, -6)$

area above curve

$$= -\int_1^5 (x^2 - 5x - 6) \, dx$$

$$= -\left[\frac{1}{3}x^3 - \frac{5}{2}x^2 - 6x\right]_1^5$$

$$= -\left[\left(\frac{125}{3} - \frac{125}{2} - 30\right) - \left(\frac{1}{3} - \frac{5}{2} - 6\right)\right]$$

$$= 42\frac{2}{3}$$

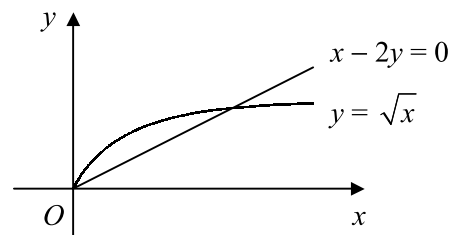
area above line

$$= \frac{1}{2} \times 4 \times (10 + 6) = 32$$

area between line and curve

$$= 42\frac{2}{3} - 32 = 10\frac{2}{3}$$

d $x - 2y = 0 \Rightarrow y = \frac{1}{2}x$



$$x^{\frac{1}{2}} = \frac{1}{2}x$$

$$\frac{1}{2}x^{\frac{1}{2}}(2 - x^{\frac{1}{2}}) = 0$$

$$x^{\frac{1}{2}} = 0, 2$$

$$x = 0, 4$$

\therefore intersect at $(0, 0)$ and $(4, 2)$

area below curve

$$= \int_0^4 x^{\frac{1}{2}} \, dx$$

$$= \left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^4$$

$$= \frac{16}{3} - 0 = \frac{16}{3}$$

area below line

$$= \frac{1}{2} \times 4 \times 2 = 4$$

area between line and curve

$$= \frac{16}{3} - 4 = \frac{4}{3}$$