

C1 DIFFERENTIATION

Worksheet C

- 1 Find the gradient at the point with x -coordinate 3 on each of the following curves.
- a** $y = x^3$ **b** $y = 4x - x^2$ **c** $y = 2x^2 - 8x + 3$ **d** $y = \frac{3}{x} + 2$
- 2 Find the gradient of each curve at the given point.
- a** $y = 3x^2 + x - 5$ (1, -1) **b** $y = x^4 + 2x^3$ (-2, 0)
- c** $y = x(2x - 3)$ (2, 2) **d** $y = x^2 - 2x^{-1}$ (2, 3)
- e** $y = x^2 + 6x + 8$ (-3, -1) **f** $y = 4x + x^{-2}$ ($\frac{1}{2}$, 6)
- 3 Evaluate $f'(4)$ when
- a** $f(x) = (x + 1)^2$ **b** $f(x) = x^{\frac{1}{2}}$ **c** $f(x) = x - 4x^{-2}$ **d** $f(x) = 5 - 6x^{\frac{3}{2}}$
- 4 The curve with equation $y = x^3 - 4x^2 + 3x$ crosses the x -axis at the points A , B and C .
- a** Find the coordinates of the points A , B and C .
- b** Find the gradient of the curve at each of the points A , B and C .
- 5 For the curve with equation $y = 2x^2 - 5x + 1$,
- a** find $\frac{dy}{dx}$,
- b** find the value of x for which $\frac{dy}{dx} = 7$.
- 6 Find the coordinates of the points on the curve with the equation $y = x^3 - 8x$ at which the gradient of the curve is 4.
- 7 A curve has the equation $y = x^3 + x^2 - 4x + 1$.
- a** Find the gradient of the curve at the point $P(-1, 5)$.
- Given that the gradient at the point Q on the curve is the same as the gradient at the point P ,
- b** find, as exact fractions, the coordinates of the point Q .
- 8 Find an equation of the tangent to each curve at the given point.
- a** $y = x^2$ (2, 4) **b** $y = x^2 + 3x + 4$ (-1, 2)
- c** $y = 2x^2 - 6x + 8$ (1, 4) **d** $y = x^3 - 4x^2 + 2$ (3, -7)
- 9 Find an equation of the tangent to each curve at the given point. Give your answers in the form $ax + by + c = 0$, where a , b and c are integers.
- a** $y = 3 - x^2$ (-3, -6) **b** $y = \frac{2}{x}$ (2, 1)
- c** $y = 2x^2 + 5x - 1$ ($\frac{1}{2}$, 2) **d** $y = x - 3\sqrt{x}$ (4, -2)
- 10 Find an equation of the normal to each curve at the given point. Give your answers in the form $ax + by + c = 0$, where a , b and c are integers.
- a** $y = x^2 - 4$ (1, -3) **b** $y = 3x^2 + 7x + 7$ (-2, 5)
- c** $y = x^3 - 8x + 4$ (2, -4) **d** $y = x - \frac{6}{x}$ (3, 1)

C1 DIFFERENTIATION*Worksheet C continued*

- 11** Find, in the form $y = mx + c$, an equation of
- the tangent to the curve $y = 3x^2 - 5x + 2$ at the point on the curve with x -coordinate 2,
 - the normal to the curve $y = x^3 + 5x^2 - 12$ at the point on the curve with x -coordinate -3 .
- 12** A curve has the equation $y = x^3 + 3x^2 - 16x + 2$.
- Find an equation of the tangent to the curve at the point $P(2, -10)$.
The tangent to the curve at the point Q is parallel to the tangent at the point P .
 - Find the coordinates of the point Q .
- 13** A curve has the equation $y = x^2 - 3x + 4$.
- Find an equation of the normal to the curve at the point $A(2, 2)$.
The normal to the curve at A intersects the curve again at the point B .
 - Find the coordinates of the point B .
- 14** $f(x) \equiv x^3 + 4x^2 - 18$.
- Find $f'(x)$.
 - Show that the tangent to the curve $y = f(x)$ at the point on the curve with x -coordinate -3 passes through the origin.
- 15** The curve C has the equation $y = 6 + x - x^2$.
- Find the coordinates of the point P , where C crosses the positive x -axis, and the point Q , where C crosses the y -axis.
 - Find an equation of the tangent to C at P .
 - Find the coordinates of the point where the tangent to C at P meets the tangent to C at Q .
- 16** The straight line l is a tangent to the curve $y = x^2 - 5x + 3$ at the point A on the curve.
Given that l is parallel to the line $3x + y = 0$,
- find the coordinates of the point A ,
 - find the equation of the line l in the form $y = mx + c$.
- 17** The line with equation $y = 2x + k$ is a normal to the curve with equation $y = \frac{16}{x^2}$.
Find the value of the constant k .
- 18** A ball is thrown vertically downwards from the top of a cliff. The distance, s metres, of the ball from the top of the cliff after t seconds is given by $s = 3t + 5t^2$.
Find the rate at which the distance the ball has travelled is increasing when
- $t = 0.6$,
 - $s = 54$.
- 19** Water is poured into a vase such that the depth, h cm, of the water in the vase after t seconds is given by $h = kt^{\frac{1}{3}}$, where k is a constant. Given that when $t = 1$, the depth of the water in the vase is increasing at the rate of 3 cm per second,
- find the value of k ,
 - find the rate at which h is increasing when $t = 8$.