

Paper Reference(s)

6666/01

**Edexcel GCE
Core Mathematics C4
Advanced Level**

**Tuesday 22 January 2008 – Afternoon
Time: 1 hour 30 minutes**

Materials required for examination

Mathematical Formulae (Green)

Items included with question papers

Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1.

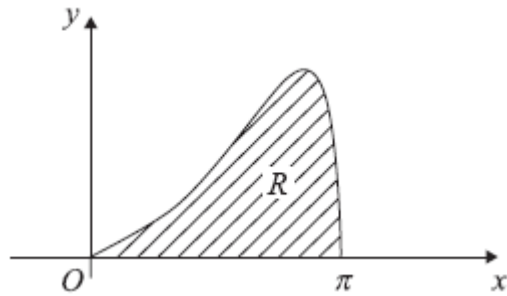


Figure 1

The curve shown in Figure 1 has equation $e^x\sqrt{\sin x}$, $0 \leq x \leq \pi$. The finite region R bounded by the curve and the x -axis is shown shaded in Figure 1.

- (a) Copy and complete the table below with the values of y corresponding to $x = \frac{\pi}{4}$ and $x = \frac{\pi}{2}$, giving your answers to 5 decimal places.

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0			8.87207	0

(2)

- (b) Use the trapezium rule, with all the values in the completed table, to obtain an estimate for the area of the region R . Give your answer to 4 decimal places.

(4)

2. (a) Use the binomial theorem to expand

$$(8 - 3x)^{\frac{1}{3}}, \quad |x| < \frac{8}{3},$$

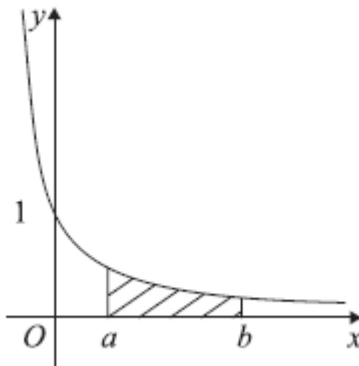
in ascending powers of x , up to and including the term in x^3 , giving each term as a simplified fraction.

(5)

- (b) Use your expansion, with a suitable value of x , to obtain an approximation to $\sqrt[3]{7.7}$. Give your answer to 7 decimal places.

(2)

3.



The curve shown in Figure 2 has equation $y = \frac{1}{(2x+1)}$. The finite region bounded by the curve, the x -axis and the lines $x = a$ and $x = b$ is shown shaded in Figure 2. This region is rotated through 360° about the x -axis to generate a solid of revolution.

Find the volume of the solid generated. Express your answer as a single simplified fraction, in terms of a and b .

(5)

4. (i) Find $\int \ln\left(\frac{x}{2}\right) dx$.

(4)

(ii) Find the exact value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \, dx$.

(5)

5. A curve is described by the equation

$$x^3 - 4y^2 = 12xy.$$

(a) Find the coordinates of the two points on the curve where $x = -8$.

(3)

(b) Find the gradient of the curve at each of these points.

(6)

6. The points A and B have position vectors $2\mathbf{i} + 6\mathbf{j} - \mathbf{k}$ and $3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ respectively.

The line l_1 passes through the points A and B .

(a) Find the vector \overrightarrow{AB} . (2)

(b) Find a vector equation for the line l_1 . (2)

A second line l_2 passes through the origin and is parallel to the vector $\mathbf{i} + \mathbf{k}$. The line l_1 meets the line l_2 at the point C .

(c) Find the acute angle between l_1 and l_2 . (3)

(d) Find the position vector of the point C . (4)

7.

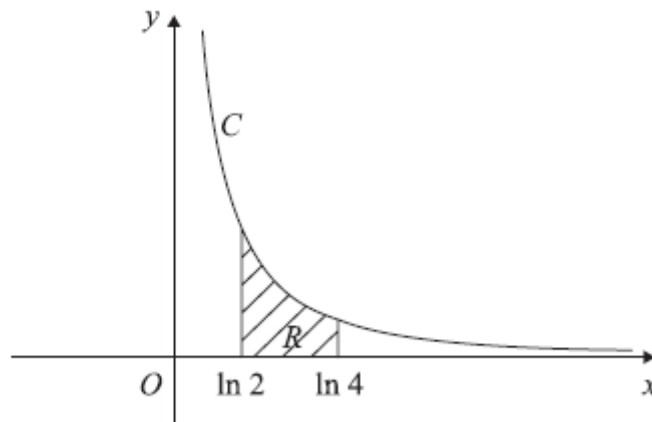


Figure 3

The curve C has parametric equations

$$x = \ln(t + 2), \quad y = \frac{1}{(t + 1)}, \quad t > -1.$$

The finite region R between the curve C and the x -axis, bounded by the lines with equations $x = \ln 2$ and $x = \ln 4$, is shown shaded in Figure 3.

(a) Show that the area of R is given by the integral

$$\int_0^2 \frac{1}{(t + 1)(t + 2)} dt. \tag{4}$$

(b) Hence find an exact value for this area.

(6)

(c) Find a cartesian equation of the curve C , in the form $y = f(x)$.

(4)

(d) State the domain of values for x for this curve.

(1)

8. Liquid is pouring into a large vertical circular cylinder at a constant rate of $1600 \text{ cm}^3\text{s}^{-1}$ and is leaking out of a hole in the base, at a rate proportional to the square root of the height of the liquid already in the cylinder. The area of the circular cross section of the cylinder is 4000 cm^2 .

- (a) Show that at time t seconds, the height h cm of liquid in the cylinder satisfies the differential equation

$$\frac{dh}{dt} = 0.4 - k\sqrt{h},$$

where k is a positive constant.

(3)

When $h = 25$, water is leaking out of the hole at $400 \text{ cm}^3\text{s}^{-1}$.

- (b) Show that $k = 0.02$.

(1)

- (c) Separate the variables of the differential equation

$$\frac{dh}{dt} = 0.4 - 0.02\sqrt{h}$$

to show that the time taken to fill the cylinder from empty to a height of 100 cm is given by

$$\int_0^{100} \frac{50}{20 - \sqrt{h}} dh.$$

(2)

Using the substitution $h = (20 - x)^2$, or otherwise,

- (d) find the exact value of $\int_0^{100} \frac{50}{20 - \sqrt{h}} dh$.

(6)

- (e) Hence find the time taken to fill the cylinder from empty to a height of 100 cm, giving your answer in minutes and seconds to the nearest second.

(1)

TOTAL FOR PAPER: 75 MARKS

END