Cubics - Past Edexcel Exam Questions

1. Factorise completely

\[ x^3 - 4x^2 + 3x \]

[3]

Question 1 - Jan 2006

2. Given that \( f(x) = (x^2 - 6x)(x - 2) + 3x \),

(a) express \( f(x) \) in the form \( x(ax^2 + bx + c) \), where \( a \), \( b \) and \( c \) are constants. [3]

(b) Hence factorise \( f(x) \) completely. [2]

(c) Sketch the graph of \( y = f(x) \), showing the coordinates of each point at which the graph meets the axes. [3]

Question 9 - May 2006

3. (a) On the same axes sketch the graphs of the curves with equations

i. \( y = x^2(x - 2) \) [3]

ii. \( y = x(6 - x) \) [3]

and indicate on your sketches the coordinates of all the points where the curves cross the \( x \)-axis.

(b) Use algebra to find the coordinates of the points where the graphs intersect. [7]

Question 10 - Jan 2007

4. The curve \( C \) has equation

\[ y = (x + 3)(x - 1)^2 \]

(a) Sketch \( C \), showing clearly the coordinates of the points where the curve meets the coordinate axes. [4]
(b) Show that the equation of $C$ can be written in the form

$$y = x^3 + x^2 - 5x + k,$$

where $k$ is a positive integer, and state the value of $k$. [2]

There are two points on $C$ where the gradient of the tangent to $C$ is equal to 3.

(c) Find the $x$-coordinates of these two points. [6]

Question 10 - Jan 2008

5. Factorise completely

$$x^3 - 9x$$

[3]

Question 2 - Jun 2008

6. The point $P(1, a)$ lies on the curve with equation $y = (x + 1)^2(2 - x)$.

(a) Find the value of $a$. [1]

(b) On the axes below, sketch the curves with the following equations:

i. $y = (x + 1)^2(2 - x),$

ii. $y = \frac{2}{x}$.

On your diagram show clearly the coordinates of any points at which the curves meet the axes. [5]
(c) With reference to your diagram in part (b), state the number of real solutions to
the equation

\[(x + 1)^2(2 - x) = \frac{2}{x}\]

[1]

Question 8 - Jan 2009

7. (a) Factorise completely \(x^3 - 6x^2 + 9x\). [3]

(b) Sketch the curve with equation

\[y = x^3 - 6x^2 + 9x\]

showing the coordinates of the points at which the curve meets the \(x\)-axis. [4]

Using your answer to part (b), or otherwise,

(c) sketch, on a separate diagram, the curve with equation

\[y = (x - 2)^2 - 6(x - 2)^2 + 9(x - 2)\]

showing the coordinates of the points at which the curve meets the \(x\)-axis. [2]

Question 10 - Jun 2009

8. (a) Factorise completely \(x^3 - 4x\). [3]

(b) Sketch the curve with equation

\[y = x^3 - 4x,\]

showing the coordinates of the points at which the curve meets the \(x\)-axis. [3]

The point \(A\) with \(x\)-coordinate -1 and the point \(B\) with \(x\)-coordinate 3 lie on the curve \(C\).

(c) Find an equation of the line which passes through \(A\) and \(B\), giving your answer in the form \(y = mx + c\), where \(m\) and \(c\) are constants. [5]
(d) Show that the length of $AB$ is $k\sqrt{10}$, where $k$ is a constant to be found.  \[2\]

Question 9 - Jan 2010

9. (a) On the axes below sketch the graphs of
i. $y = x(4 - x)$,  
ii. $y = x^2(7 - x)$,
showing clearly the coordinates of the points where the curves cross the coordinate axes. \[5\]

(b) Show that the $x$-coordinates of the points of intersection of

$$y = x(4 - x) \quad \text{and} \quad y = x^2(7 - x)$$

are given by the solutions to the equation $x(x^2 - 8x + 4) = 0$. \[3\]

The point $A$ lies on both the curves and the $x$ and $y$ coordinates of $A$ are both positive.

(c) Find the exact coordinates of $A$, leaving your answer in the form $(p + q\sqrt{3}, r + s\sqrt{3})$, where $p$, $q$, $r$ and $s$ are integers. \[7\]

Question 10 - May 2010

10. (a) Sketch the graphs of
i. $y = x(x + 2)(3 - x)$,  
ii. $y = -\frac{2}{x}$,
showing clearly the coordinates of all the point where the curves cross the coordinate axes. \[6\]
(b) Using your sketch state, giving a reason, the number of real solutions to the equation

\[ x(x + 2)(3 - x) + \frac{2}{x} = 0 \]

[2]

Question 10 - Jan 2011
Solutions

1. \( x(x - 3)(x - 1) \)

2. (a) \( x(x^2 - 8x + 15) \)
   (b) \( x(x - 5)(x - 3) \)
   (c) \( y = x(x - 5)(x - 3) \)

3. (a) \( y = x^2(x - 2) \)
   (b) \( y = x(6 - x) \)
   (c) \( (-2, -16), (3, 9) \)
4. (a) 
\[ y = (x + 3) (x - 1)^2 \]

(b) \( k = 3 \)

(c) \( x = \frac{4}{3}, \ x = -2 \)

5. \( x(x - 3)(x + 3) \)

6. (a) \( a = 4 \)

(b) 
\[ y = (x + 1)^2 (2 - x) \]

(c) The graphs intersect twice and so there are 2 solutions. We know they intersect twice since the point (1,4), on the reciprocal functions, lies above the point (1,2) on the cubic.
7. (a) \( x(x - 3)^2 \)  
   (b) 
   \[ y = x(x - 3)^2 \] 
   (0, 0) (3, 0) 
   \[ y = x(x - 3)^2 \]  
   (c) 

8. (a) \( x(x - 2)(x + 2) \)  
   (b) 
   \[ y = x(x - 2)(x + 2) \]  
   (2, 0) (5, 0) 
   \[ y = (x - 2)(x - 5)^2 \]  
   (-50, 0) 
   (c) \( y = 3x + 6 \)
(d) \( k = 4 \)

9. (a) 

\[ y = x^2(7 - x) \]

\[ y = x(4 - x) \]

(b) -

(c) \((4 - 2\sqrt{3}, 8\sqrt{3} - 12)\). Note that both \(4 + 2\sqrt{3}\) and \(4 - 2\sqrt{3}\) are both positive.

10. (a) 

\[ y = x(x + 2)(3 - x) \]

\[ y = -\frac{x}{2} \]

(b) There are 2 solutions since the curves intersect twice.