Differentiation - Past Edexcel Exam Questions

1. (a) Given that \( y = 5x^3 + 7x + 3 \), find
   i. \( \frac{dy}{dx} \) [3]
   ii. \( \frac{d^2y}{dx^2} \) [1]

   Question 2ai, 2a(ii) - January 2005

2. The curve \( C \) has equation \( y = 4x^2 + \frac{5-x}{x} \), \( x \neq 0 \). The point \( P \) on \( C \) has \( x \)-coordinate 1.
   (a) Show that the value of \( \frac{dy}{dx} \) at \( P \) is 3. [5]
   (b) Find an equation of the tangent to \( C \) at \( P \). [3]
   This tangent meets the \( x \)-axis at the point \((k, 0)\).
   (c) Find the value of \( k \). [2]

   Question 7 - January 2005

3. Given that \( y = 6x - \frac{4}{x^2} \), \( x \neq 0 \),
   (a) find \( \frac{dy}{dx} \). [2]

   Question 2a - May 2005

4. The curve \( C \) has equation \( y = \frac{1}{3}x^3 - 4x^2 + 8x + 3 \).
   The point \( P \) has coordinates \((3, 0)\).
   (a) Show that \( P \) lies on \( C \). [1]
   (b) Find the equation of the tangent to \( C \) at \( P \), giving your answer in the form \( y = mx + c \), where \( m \) and \( c \) are constants. [5]

   Another point \( Q \) also lies on \( C \). The tangent to \( C \) at \( Q \) is parallel to the tangent to \( C \) at \( P \).
(c) Find the coordinates of $Q$.  

**Question 10 - May 2005**

5. Given that $y = 2x^2 - 6$, $x \neq 0$,

(a) find $\frac{dy}{dx}$.

**Question 4 - January 2006**

6. .

![Figure 1:](image)

Figure 1 shows part of the curve $C$ with equation

$$y = (x - 1)(x^2 - 4).$$

The curve cuts the $x$-axis at the points $P$, $(1,0)$ and $Q$ as shown in Figure 1.

(a) Write down the $x$-coordinate of $P$ and the $x$-coordinate of $Q$.  
(b) Show that $\frac{dy}{dx} = 3x^2 - 2x - 4$.  
(c) Show that $y = x + 7$ is an equation of the tangent to $C$ at the point $(-1,6)$.  
(d) Find the exact coordinates of $R$.  

The tangent to $C$ at the point $R$ is parallel to the tangent at the point $(-1,6)$.  

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Question 9 - January 2006

7. Differentiate with respect to $x$
   
   (a) $x^4 + 6\sqrt{x}$, [3]
   
   (b) $\frac{(x+4)^2}{x}$. [4]

Question 5 - May 2006

8. Given that $y = 4x^3 - 1 + 2x^{\frac{1}{2}}$, $x > 0$, find $\frac{dy}{dx}$. [4]

Question 1 - January 2007

9. The curve $C$ has equation $y = 4x + 3x^{\frac{3}{2}} - 2x^2$, $x > 0$.
   
   (a) Find an expression for $\frac{dy}{dx}$. [3]
   
   (b) Show that the point $P(4,8)$ lies on $C$. [1]
   
   (c) Show that an equation of the normal to $C$ at the point $P$ is $3y = x + 20$. [4]

   The normal to $C$ at $P$ cuts the $x$-axis at the point $Q$.
   
   (d) Find the length $PQ$, giving your answer in a simplified surd form. [3]

Question 8 - January 2007

10. Given that $y = 3x^2 + 4\sqrt{x}$, $x > 0$, find
    
    (a) $\frac{dy}{dx}$, [2]
    
    (b) $\frac{d^2y}{dx^2}$. [2]
11. The curve \( C \) has equation \( y = x^2(x - 6) + \frac{4}{x}, \ x > 0 \).

The points \( P \) and \( Q \) lie on \( C \) and have \( x \)-coordinates 1 and 2 respectively.

(a) Show that the length of \( PQ \) is \( \sqrt{170} \). \[4\]

(b) Show that the tangents to \( C \) at \( P \) and \( Q \) are parallel. \[5\]

(c) Find an equation for the normal to \( C \) at \( P \), giving your answer in the form \( ax + by + c = 0 \), where \( a \), \( b \) and \( c \) are integers. \[4\]

Question 10 - May 2007

12. (a) Write \( \frac{2\sqrt{x} + 3}{x} \) in the form \( 2x^p + 3x^q \), where \( p \) and \( q \) are constants. \[2\]

Given that \( y = 5x - 7 + \frac{2\sqrt{x} + 3}{x}, \ x > 0 \),

(b) find \( \frac{dy}{dx} \), simplifying the coefficient of each term. \[4\]

Question 5 - January 2008

13.

\[ f(x) = 3x + x^3, \ x > 0. \]

(a) Differentiate to find \( f'(x) \). \[2\]

Given that \( f'(x) = 15 \),

(b) find the value of \( x \). \[3\]

Question 4 - June 2008

14. The curve \( C \) has equation \( y = kx^3 - x^2 + x - 5 \), where \( k \) is a constant.
Differentiation

(a) Find \( \frac{dy}{dx} \).

The point \( A \) with \( x \)-coordinate \( -\frac{1}{2} \) lies on \( C \). The tangent to \( C \) at \( A \) is parallel to the line with equation \( 2y - 7x + 1 = 0 \).

Find

(b) the value of \( k \),

(c) the value of the \( y \)-coordinate of \( A \).

Question 9 - June 2008

15. Given that \( \frac{2x^2-x^\frac{3}{2}}{\sqrt{x}} \) can be written in the form \( 2x^p - x^q \),

(a) write down the value of \( p \) and the value of \( q \).

Given that \( y = 5x^4 - 3 + \frac{2x^2-x^\frac{3}{2}}{\sqrt{x}} \), \( x > 0 \),

(b) find \( \frac{dy}{dx} \), simplifying the coefficient of each term.

Question 6 - January 2009

16. The curve \( C \) has equation

\[ y = 9 - 4x - \frac{8}{x}, \quad x > 0. \]

The point \( P \) on \( C \) has \( x \)-coordinate equal to 2.

(a) Show that the equation of the tangent to \( C \) at the point \( P \) is \( y = 1 - 2x \).

(b) Find an equation of the normal to \( C \) at the point \( P \).

The tangent at \( P \) meets the \( x \)-axis at \( A \) and the normal at \( P \) meets the \( x \)-axis at \( B \).

(c) Find the area of the triangle \( APB \).

Question 11 - January 2009

17. Given that \( y = 2x^3 + \frac{3}{x^2}, \ x \neq 0 \), find
18. \[ f(x) = \frac{(3 - 4\sqrt{x})^2}{\sqrt{x}}, \quad x > 0 \]

(a) Show that \( f(x) = 9x^{-\frac{1}{2}} + Ax^\frac{1}{2} + B \), where \( A \) and \( B \) are constants to be found.

(b) Find \( f'(x) \).

(c) Evaluate \( f'(9) \).

Question 9 - June 2009

19. The curve \( C \) has equation

\[ y = x^3 - 2x^2 - x + 9, \quad x > 0 \]

The point \( P \) has coordinates \((2,7)\).

(a) Show that \( P \) lies on \( C \).

(b) Find the equation of the tangent to \( C \) at \( P \), giving your answer in the form \( y = mx + c \), where \( m \) and \( c \) are constants.

The point \( Q \) also lies on \( C \).

Given that the tangent to \( C \) at \( Q \) is perpendicular to the tangent to \( C \) at \( P \),

(c) show that the \( x \)-coordinate of \( Q \) is \( \frac{1}{3}(2 + \sqrt{6}) \).

Question 11 - June 2009

20. Given that \( y = x^4 + x^\frac{1}{2} + 3 \), find \( \frac{dy}{dx} \).

Question 1 - January 2010
21. The curve \( C \) has equation
\[
y = \frac{(x+3)(x-8)}{x}, \quad x > 0.
\]
(a) Find \( \frac{dy}{dx} \) in its simplest form. \([4]\]
(b) Find an equation of the tangent to \( C \) at the point where \( x = 2 \). \([4]\]

Question 6 - January 2010

22. Given that
\[
y = 8x^3 - 4\sqrt{x} + \frac{3x^2 + 2}{x}, \quad x > 0,
\]
find \( \frac{dy}{dx} \). \([6]\]

Question 7 - May 2010

23. The curve \( C \) has equation
\[
y = \frac{1}{2}x^3 - 9x^{\frac{3}{2}} + \frac{8}{x} + 30, \quad x > 0.
\]
(a) Find \( \frac{dy}{dx} \). \([4]\]
(b) Show that the point \( P(4, -8) \) lies on \( C \). \([2]\]
(c) Find an equation of the normal to \( C \) at the point \( P \), giving your answer in the form \( ax + by + c = 0 \), where \( a \), \( b \) and \( c \) are integers. \([6]\]

Question 11 - January 2011

24. Given that \( y = 2x^5 + 7 + \frac{1}{x^4}, \quad x \neq 0 \), find, in their simplest form,
(a) \( \frac{dy}{dx} \). \([3]\]

Question 2a - May 2011
25. The curve $C$ has equation

$$y = (x + 1)(x + 3)^2.$$  

(a) Sketch $C$, showing the coordinates of the points at which $C$ meets the axes.  

(b) Show that $\frac{dy}{dx} = 3x^2 + 14x + 15$.  

The point $A$, with $x$-coordinate -5, lies on $C$.  

(c) Find the equation of the tangent to $C$ at $A$, giving your answer in the form $y = mx + c$, where $m$ and $c$ are constants.  

Another point $B$ also lies on $C$. The tangents to $C$ at $A$ and $B$ are parallel.  

(d) Find the $x$-coordinate of $B$.  

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26. Given that $y = x^4 + 6x^{\frac{3}{2}}$, find in their simplest form

(a) $\frac{dy}{dx}$,  

Question 10 - June 2011

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27. The curve $C_1$ has equation

$$y = x^2(x + 2)$$

(a) Find $\frac{dy}{dx}$.  

(b) Sketch $C_1$, showing the coordinates of the points where $C_1$ meets the $x$-axis.  

The curve $C_2$ has equation

$$y = (x - k)^2(x - k + 2)$$

where $k$ is a constant and $k > 2$.  

(c) Find the gradient of $C_1$ at each point where $C_1$ meets the $x$-axis.  

(d) Sketch $C_2$, showing the coordinates of the points where $C_2$ meets the $x$ and $y$ axes.  

Question 1a - January 2012
28. \[ y = 5x^3 - 6x^4 + 2x - 3. \]

(a) Find \( \frac{dy}{dx} \), giving each term in its simplest form. [4]

(b) Find \( \frac{d^2y}{dx^2} \). [2]

Question 4 - May 2012

29. The curve \( C \) has equation

\[ y = 2x - 8\sqrt{x} + 5, \quad x \geq 0 \]

(a) Find \( \frac{dy}{dx} \), giving each term in its simplest form. [3]

The point \( P \) on \( C \) has \( x \)-coordinate equal to \( \frac{1}{4} \).

(b) Find the equation of the tangent to \( C \) at the point \( P \), giving your answer in the form \( y = ax + b \), where \( a \) and \( b \) are constants. [4]

The tangent to \( C \) at the point \( Q \) is parallel to the line with equation \( 2x - 3y + 18 = 0 \).

(c) Find the coordinates of \( Q \). [5]

Question 11 - January 2013

30. \[ f'(x) = \frac{(3 - x^2)^2}{x^2}, \quad x \neq 0 \]

(a) Show that

\[ f'(x) = 9x^{-2} + A + Bx^2, \]

where \( A \) and \( B \) are constants to be found. [3]

(b) Find \( f''(x) \). [2]
31. Differentiate with respect to $x$, giving each answer in its simplest form.

(a) $(1 - 2x)^2$  
(b) $\frac{x^3 + 6\sqrt{x}}{2x^4}$

Question 7 - May 2014
Solutions

1. (a) i. $15x^2 + 7$
   ii. $30x$
2. (a) -
   (b) $y = 3x + 5$
   (c) $k = \frac{-5}{3}$
3. (a) $6 + 8x^{-3}$
4. (a) -
   (b) $y = -7x + 21$
   (c) $(5, -\frac{46}{3})$
5. (a) $4x + 18x^{-4}$
6. (a) -2, 2
   (b) -
   (c) -
   (d) $\left(\frac{5}{3}, -\frac{22}{27}\right)$
7. (a) $4x^3 + 3x^{-\frac{1}{2}}$
   (b) $1 - 16x^{-2}$
8. $12x^2 + x^{-\frac{1}{2}}$
9. (a) $4 + \frac{9}{2}x^{\frac{1}{2}} - 4x$
   (b) -
   (c) -
   (d) $8\sqrt{10}$
10. (a) $6x + 2x^{-\frac{1}{2}}$
    (b) $6 - x^{-\frac{3}{2}}$
11. (a) -
    (b) -
    (c) $x - 13y - 14 = 0$
12. (a) \(2x^{-\frac{1}{2}} + 3x^{-1}\)
   (b) \(5 - x^{-\frac{3}{2}} - 3x^{-2}\)

13. (a) \(3 + 3x^2\)
   (b) \(x = 2\)

14. (a) \(3kx^2 - 2x + 1\)
   (b) \(k = 2\)
   (c) \(-6\)

15. (a) \(p = \frac{3}{2}, q = 1\)
   (b) \(20x^3 + 3x^{\frac{1}{2}} - 1\)

16. (a) 
   (b) \(y = \frac{1}{2}x - 4\)
   (c) \(\frac{45}{4}\)

17. (a) \(6x^2 - 6x^{-3}\)

18. (a) \(A = 16, B = -24\)
   (b) \(-\frac{9}{2}x^{-\frac{3}{2}} + 8x^{-\frac{1}{2}}\)
   (c) \(\frac{5}{2}\)

19. (a) 
   (b) \(y = 3x + 1\)
   (c) 

20. \(4x^2 + \frac{1}{3}x^{-\frac{2}{3}}\)

21. (a) \(1 + 24x^{-2}\)
   (b) \(y = 7x - 29\)

22. \(24x^2 - 2x^{-\frac{1}{2}} + 3 - 2x^{-2}\)

23. (a) \(\frac{3}{2}x^2 - \frac{27}{4}x^\frac{1}{2} - 8x^{-2}\)
   (b) 
   (c) \(2x - 7y - 64 = 0\)

24. \(10x^4 - 3x^{-4}\)
25. (a) 

\[ y = (x + 1)(x + 3)^2 \]

(b) -

(c) \[ y = 20x + 84 \]

(d) \[ x = \frac{1}{3} \]

26. \[ 4x^3 + 3x^{-\frac{1}{2}} \]

27. (a) \[ 3x^2 + 4x \]

(b) .

(c) 4, 0
Differentiation

(d).

\[ y = (x - k)^2(x - k + 2) \]

28. (a) \( 15x^2 - 8x^{\frac{1}{3}} + 2 \)
    (b) \( 30x - \frac{8}{3}x^{-\frac{2}{3}} \)

29. (a) \( 2 - 4x^{-\frac{1}{4}} \)
    (b) \( y = -6x + 3 \)
    (c) \( (9, -1) \)

30. (a) \( A = -6, B = 1 \)
    (b) \( -18x^{-3} + 2x \)

31. (a) \( -4 + 8x \)
    (b) \( \frac{3}{2}x^2 - \frac{9}{2}x^{-\frac{5}{3}} \)