Sigma Notation & Sequences - Past Edexcel Exam Questions

1. The sequence of positive numbers $u_1, u_2, u_3, \ldots$, is given by

$$u_{n+1} = (u_n - 3)^2, \quad u_1 = 1$$

(a) Find $u_2$, $u_3$ and $u_4$. [3]
(b) Write down the value of $u_{20}$ [1]

Question 2 - Jan 2006

2. A sequence $a_1, a_2, a_3, \ldots$, is defined by

$$a_1 = 3,$$
$$a_{n+1} = 3a_n - 5, \quad n \geq 1$$

(a) Find the value of $a_2$ and the value of $a_3$. [2]
(b) Calculate the value of $\sum_{r=1}^{5} a_r$. [3]

Question 4 - May 2006

3. A sequence $a_1, a_2, a_3, \ldots$, is defined by

$$a_1 = k,$$
$$a_{n+1} = 3a_n + 5, \quad n \geq 1$$

where $k$ is a positive integer.

(a) Write down an expression for $a_2$ in terms of $k$. [1]
(b) Show that $a_3 = 9k + 20$. [2]
(c) i. Find $\sum_{r=1}^{4} a_r$ in terms of $k$. [3]
    ii. Show that $\sum_{r=1}^{4} a_r$ is divisible by 10. [4]
4. A sequence is given by

\[
\begin{align*}
x_1 &= 1, \\
x_{n+1} &= x_n(p + x_n),
\end{align*}
\]

where \( p \) is a constant (\( p \neq 0 \)).

(a) Find \( x_2 \) in terms of \( p \). \[1\]
(b) Show that \( x_3 = 1 + 3p + 2p^2 \). \[2\]

Given that \( x_3 = 1 \),
(c) find the value of \( p \). \[3\]
(d) write down the value of \( x_{2008} \). \[2\]

Question 7 - Jan 2008

5. A sequence \( x_1, x_2, x_3, \ldots \), is defined by

\[
\begin{align*}
x_1 &= 1, \\
x_{n+1} &= ax_n - 3, \quad n \geq 1
\end{align*}
\]

where \( a \) is a constant.

(a) Find an expression for \( x_2 \) in terms of \( a \). \[1\]
(b) Show that \( x_3 = a^2 - 3a - 3 \). \[2\]

Given that \( x_3 = 7 \),
(c) find the possible values of \( a \). \[3\]
6. A sequence \( a_1, a_2, a_3, \ldots \), is defined by

\[
\begin{align*}
a_1 &= k, \\
a_{n+1} &= 2a_n - 7, \quad n \geq 1
\end{align*}
\]

where \( k \) is a constant.

(a) Write down an expression for \( a_2 \) in terms of \( k \). [1]

(b) Show that \( a_3 = 4k - 21 \). [2]

Given that \( \sum_{r=1}^{4} a_r = 43 \),

(c) find the value of \( k \). [4]

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7. A sequence of positive numbers is defined by

\[
\begin{align*}
a_{n+1} &= \sqrt{a_n^2 + 3}, \quad n \geq 1, \\
a_1 &= 2.
\end{align*}
\]

(a) Find \( a_2 \) and \( a_3 \), leaving your answers in surd form. [2]

(b) Show that \( a_5 = 4 \). [2]

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8. A sequence \( a_1, a_2, a_3, \ldots \), is defined by

\[
\begin{align*}
a_1 &= 2, \\
a_{n+1} &= 3a_n - c, \quad n \geq 1
\end{align*}
\]

where \( c \) is a constant.

(a) Find an expression for \( a_2 \) in terms of \( c \). [1]

Given that \( \sum_{i=1}^{3} a_i = 0 \),

(b) find the value of \( c \). [4]
Question 4 - Jan 2011

9. A sequence \(a_1, a_2, a_3, \ldots\), is defined by

\[
\begin{align*}
    a_1 &= k, \\
    a_{n+1} &= 5a_n + 3, & n \geq 1
\end{align*}
\]

where \(k\) is a positive integer.

(a) Write down an expression for \(a_2\) in terms of \(k\).  
(b) Show that \(a_3 = 25k + 18\).

(c) i. Find \(\sum_{r=1}^{4} a_r\) in terms of \(k\), in its simplest form.
    ii. Show that \(\sum_{r=1}^{4} a_r\) is divisible by 6.

Question 5 - May 2011

10. A sequence \(x_1, x_2, x_3, \ldots\), is defined by

\[
\begin{align*}
    x_1 &= 1, \\
    x_{n+1} &= ax_n + 5, & n \geq 1
\end{align*}
\]

where \(a\) is a constant.

(a) Write down an expression for \(x_2\) in terms of \(a\).
(b) Show that \(x_3 = a^2 + 5a + 5\).

   Given that \(x_3 = 41\),

(c) find the possible values of \(a\).
Sigma Notation Questions

11. A sequence $a_1$, $a_2$, $a_3$, ..., is defined by

\[
\begin{align*}
    a_1 &= 3, \\
    a_{n+1} &= 2a_n - c, \quad n \geq 1
\end{align*}
\]

where $c$ is a constant.

(a) Write down an expression, in terms of $c$, for $a_2$. \[1\]

(b) Show that $a_3 = 12 - 3c$. \[2\]

Given that $\sum_{i=1}^{4} a_i \geq 23$, 
(c) find the range of values of $c$. \[4\]

Question 5 - May 2012

12. A sequence $u_1$, $u_2$, $u_3$, ..., satisfies

\[
    u_{n+1} = 2u_n - 1, \quad n \geq 1
\]

Given that $u_2 = 9$,

(a) find the value of $u_3$ and the value of $u_4$, \[2\]

(b) evaluate $\sum_{r=1}^{4} u_r$. \[3\]

Question 4 - Jan 2013

13. A sequence $a_1$, $a_2$, $a_3$, ... is defined by

\[
\begin{align*}
    a_1 &= 4, \\
    a_{n+1} &= k(a_n + 2), \quad n \geq 1
\end{align*}
\]

where $k$ is a constant.

(a) Find an expression for $a_2$ in terms of $k$. \[1\]

Given that $\sum_{i=1}^{3} a_i = 2$, 

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14. A sequence of numbers $a_1, a_2, a_3, \ldots$, is defined by

$$a_{n+1} = 5a_n - 3, \quad n \geq 1$$

Given that $a_2 = 7$,

(a) find the value of $a_1$. \[2\]

(b) Find the value of $\sum_{r=1}^{4} a_r$. \[3\]

Question 5 - Jun 2014
Solutions

1. (a) $u_2 = 4$, $u_3 = 1$, $u_4 = 4$
   (b) $u_{20} = 4$

2. (a) $a_2 = 4$, $a_3 = 7$
   (b) $73$

3. (a) $u_2 = 3k + 5$
   (b) -
   (c) i. $40k + 90$
        ii. -

4. (a) $x_2 = p + 1$
   (b) -
   (c) $p = -\frac{3}{2}$
   (d) $x_{2008} = -\frac{1}{2}$

5. (a) $x_2 = a - 3$
   (b) -
   (c) $a = 5$, $a = -2$

6. (a) $a_2 = 2k - 7$
   (b) -
   (c) $k = 8$

7. (a) $a_2 = \sqrt{7}$, $a_3 = \sqrt{10}$
   (b) -

8. (a) $a_2 = 6 - c$
   (b) $c = \frac{26}{5}$

9. (a) $a_2 = 5k + 3$
   (b) -
   (c) $156k + 114$
   (d) -
10. (a) $x_2 = a + 5$
  (b) -
  (c) $a = 4$, $a = -9$

11. (a) $a_2 = 6 - c$
  (b) -
  (c) $c \leq 2$

12. (a) $u_3 = 17$, $u_4 = 33$
  (b) 64

13. (a) $a_2 = 6k$
  (b) $k = -1$, $k = -\frac{1}{3}$

14. (a) $a_1 = 2$
  (b) 198