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## Sigma Notation & Sequences - Past Edexcel Exam Questions

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1. The sequence of positive numbers  $u_1, u_2, u_3, \dots$ , is given by

$$u_{n+1} = (u_n - 3)^2, \quad u_1 = 1$$

- (a) Find  $u_2, u_3$  and  $u_4$ . [3]  
(b) Write down the value of  $u_{20}$ . [1]

**Question 2 - Jan 2006**

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2. A sequence  $a_1, a_2, a_3, \dots$ , is defined by

$$\begin{aligned} a_1 &= 3, \\ a_{n+1} &= 3a_n - 5, \quad n \geq 1 \end{aligned}$$

- (a) Find the value of  $a_2$  and the value of  $a_3$ . [2]  
(b) Calculate the value of  $\sum_{r=1}^5 a_r$ . [3]

**Question 4 - May 2006**

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3. A sequence  $a_1, a_2, a_3, \dots$ , is defined by

$$\begin{aligned} a_1 &= k, \\ a_{n+1} &= 3a_n + 5, \quad n \geq 1 \end{aligned}$$

where  $k$  is a positive integer.

- (a) Write down an expression for  $a_2$  in terms of  $k$ . [1]  
(b) Show that  $a_3 = 9k + 20$ . [2]  
(c) i. Find  $\sum_{r=1}^4 a_r$  in terms of  $k$ .  
ii. Show that  $\sum_{r=1}^4 a_r$  is divisible by 10. [4]

**Question 8 - May 2007**

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4. A sequence is given by

$$\begin{aligned}x_1 &= 1, \\x_{n+1} &= x_n(p + x_n),\end{aligned}$$

where  $p$  is a constant ( $p \neq 0$ ).

- (a) Find  $x_2$  in terms of  $p$ . [1]  
(b) Show that  $x_3 = 1 + 3p + 2p^2$ . [2]

Given that  $x_3 = 1$ ,

- (c) find the value of  $p$ , [3]  
(d) write down the value of  $x_{2008}$ . [2]

**Question 7 - Jan 2008**

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5. A sequence  $x_1, x_2, x_3, \dots$ , is defined by

$$\begin{aligned}x_1 &= 1, \\x_{n+1} &= ax_n - 3, \quad n \geq 1\end{aligned}$$

where  $a$  is a constant.

- (a) Find an expression for  $x_2$  in terms of  $a$ . [1]  
(b) Show that  $x_3 = a^2 - 3a - 3$ . [2]

Given that  $x_3 = 7$ ,

- (c) find the possible values of  $a$ . [3]

**Question 5 - June 2008**

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6. A sequence  $a_1, a_2, a_3, \dots$ , is defined by

$$\begin{aligned}a_1 &= k, \\a_{n+1} &= 2a_n - 7, \quad n \geq 1\end{aligned}$$

where  $k$  is a constant.

(a) Write down an expression for  $a_2$  in terms of  $k$ . [1]

(b) Show that  $a_3 = 4k - 21$ . [2]

Given that  $\sum_{r=1}^4 a_r = 43$ ,

(c) find the value of  $k$ . [4]

**Question 7 - June 2009**

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7. A sequence of positive numbers is defined by

$$\begin{aligned}a_{n+1} &= \sqrt{a_n^2 + 3}, \quad n \geq 1, \\a_1 &= 2.\end{aligned}$$

(a) Find  $a_2$  and  $a_3$ , leaving your answers in surd form. [2]

(b) Show that  $a_5 = 4$ . [2]

**Question 5 - May 2010**

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8. A sequence  $a_1, a_2, a_3, \dots$ , is defined by

$$\begin{aligned}a_1 &= 2, \\a_{n+1} &= 3a_n - c, \quad n \geq 1\end{aligned}$$

where  $c$  is a constant.

(a) Find an expression for  $a_2$  in terms of  $c$ . [1]

Given that  $\sum_{i=1}^3 a_i = 0$ ,

(b) find the value of  $c$ . [4]

**Question 4 - Jan 2011**

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9. A sequence  $a_1, a_2, a_3, \dots$ , is defined by

$$\begin{aligned}a_1 &= k, \\ a_{n+1} &= 5a_n + 3, \quad n \geq 1\end{aligned}$$

where  $k$  is a positive integer.

(a) Write down an expression for  $a_2$  in terms of  $k$ . [1]

(b) Show that  $a_3 = 25k + 18$ . [2]

(c) i. Find  $\sum_{r=1}^4 a_r$  in terms of  $k$ , in its simplest form.

ii. Show that  $\sum_{r=1}^4 a_r$  is divisible by 6. [4]

**Question 5 - May 2011**

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10. A sequence  $x_1, x_2, x_3, \dots$ , is defined by

$$\begin{aligned}x_1 &= 1, \\ x_{n+1} &= ax_n + 5, \quad n \geq 1\end{aligned}$$

where  $a$  is a constant.

(a) Write down an expression for  $x_2$  in terms of  $a$ . [1]

(b) Show that  $x_3 = a^2 + 5a + 5$ . [2]

Given that  $x_3 = 41$ ,

(c) find the possible values of  $a$ . [3]

**Question 4 - Jan 2012**

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11. A sequence  $a_1, a_2, a_3, \dots$ , is defined by

$$\begin{aligned}a_1 &= 3, \\ a_{n+1} &= 2a_n - c, \quad n \geq 1\end{aligned}$$

where  $c$  is a constant.

(a) Write down an expression, in terms of  $c$ , for  $a_2$ . [1]

(b) Show that  $a_3 = 12 - 3c$ . [2]

Given that  $\sum_{i=1}^4 a_i \geq 23$ ,

(c) find the range of values of  $c$ . [4]

**Question 5 - May 2012**

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12. A sequence  $u_1, u_2, u_3, \dots$ , satisfies

$$u_{n+1} = 2u_n - 1, \quad n \geq 1$$

Given that  $u_2 = 9$ ,

(a) find the value of  $u_3$  and the value of  $u_4$ , [2]

(b) evaluate  $\sum_{r=1}^4 u_r$ . [3]

**Question 4 - Jan 2013**

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13. A sequence  $a_1, a_2, a_3, \dots$  is defined by

$$\begin{aligned}a_1 &= 4, \\ a_{n+1} &= k(a_n + 2), \quad n \geq 1\end{aligned}$$

where  $k$  is a constant.

(a) Find an expression for  $a_2$  in terms of  $k$ . [1]

Given that  $\sum_{i=1}^3 a_i = 2$ ,

- (b) find the two possible values of  $k$ . [6]

**Question 4 - Jun 2013**

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14. A sequence of numbers  $a_1, a_2, a_3, \dots$ , is defined by

$$a_{n+1} = 5a_n - 3, \quad n \geq 1$$

Given that  $a_2 = 7$ ,

- (a) find the value of  $a_1$ . [2]
- (b) Find the value of  $\sum_{r=1}^4 a_r$ . [3]

**Question 5 - Jun 2014**

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## Solutions

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1. (a)  $u_2 = 4, u_3 = 1, u_4 = 4$

(b)  $u_{20} = 4$

2. (a)  $a_2 = 4, a_3 = 7$

(b) 73

3. (a)  $u_2 = 3k + 5$

(b) -

(c) i.  $40k + 90$

ii. -

4. (a)  $x_2 = p + 1$

(b) -

(c)  $p = -\frac{3}{2}$

(d)  $x_{2008} = -\frac{1}{2}$

5. (a)  $x_2 = a - 3$

(b) -

(c)  $a = 5, a = -2$

6. (a)  $a_2 = 2k - 7$

(b) -

(c)  $k = 8$

7. (a)  $a_2 = \sqrt{7}, a_3 = \sqrt{10}$

(b) -

8. (a)  $a_2 = 6 - c$

(b)  $c = \frac{26}{5}$

9. (a)  $a_2 = 5k + 3$

(b) -

(c)  $156k + 114$

(d) -

10. (a)  $x_2 = a + 5$   
(b) -  
(c)  $a = 4, a = -9$
11. (a)  $a_2 = 6 - c$   
(b) -  
(c)  $c \leq 2$
12. (a)  $u_3 = 17, u_4 = 33$   
(b) 64
13. (a)  $a_2 = 6k$   
(b)  $k = -1, k = -\frac{1}{3}$
14. (a)  $a_1 = 2$   
(b) 198