Proof TEST

For each of the following, either prove that the statement is true by deduction or exhaustion or disprove the statement by providing a counterexample.

1. For any $x \in \mathbb{R}$, if $x^2 > 9$ then $x > 3$.

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2. The sum of two odd numbers is always even.

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3. (a) The difference of two consecutive square numbers between 75 and 150 is odd.

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   (b) The difference of ANY two consecutive square numbers is ALWAYS odd.

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4. Given that n is a positive integer, it follows that $2^n \geq n^2$.

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5. The product of 3 consecutive even numbers is a multiple of 8.

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6. (a) If n is an integer such that $4 \leq n \leq 7$, then $3n^2 + n + 14$ is even.

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(b) If n is ANY positive integer, $3n^2 + n + 14$ is ALWAYS even.

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7. $3^x - 3$ is divisible by $x \in \mathbb{N}$ if $x > 1$.

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8. Given two positive integers, the difference of their squares is equal to their sum if the difference between them is 1.

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9. For all $m \in \mathbb{Z}$, $m^2 \geq m$.

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10. (a) If $a, b \in \mathbb{Q}$, then $a + b \in \mathbb{Q}$.

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(b) If $a + b \in \mathbb{Q}$, then $a, b \in \mathbb{Q}$.

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11. The Triangle Inequality:

$$|p + q| \leq |p| + |q|,$$

holds for all $p, q \in \mathbb{R}$. . . . .

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12. (a) (Mersenne Primes) For any prime number p, $2^p - 1$ is prime.

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(b) For any positive integer n, $2^{2^n} + 1$ is prime.

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