

## AS-Level Mathematics - Pure Maths Test - 'INTERMEDIATE'

1. (a) Factorise  $9x^2 - 4y^2$  .....
- (b) Simplify  $(4x^2)^{\frac{-3}{2}}$  .....
- .....
- (c) Rationalise the denominator of  $\frac{\sqrt{5}-2}{\sqrt{5}+3}$  .....
- .....
- .....
2. (a) Show that  $x - 2$  is a factor of  $f(x) = 2x^3 - 3x^2 - 5x + 6$  .....
- .....
- (b) Factorise  $f(x)$  completely .....
- (c) Sketch  $f(x)$ .
3. (a) Determine the set of values of  $k$  for which the equation  $x^2 + 2x - k = 0$  has 2 real solutions. ....
- .....
- (b) Suppose  $k = 2$ . By first completing the square, sketch the graph of  $y = x^2 + 2x - k$ , labelling clearly any intersection of axes and the vertex of the graph.

4. Solve the following simultaneous equations:

$$\begin{aligned} 2y + x - 3 &= 0 \\ x^2 + 3xy - 10 &= 0 \end{aligned}$$

.....  
 .....  
 .....

5. Find the values of  $x$  for which  $x^2 < 2x + 3$  .....

.....

.....

6. Sketch the graph of  $f(x) = 4x^2 - 4x - x^3$ .

7. The line  $l_1$  passes through the points  $A(2, 6)$  and  $B(0, -1)$ . The line  $l_2$  is perpendicular to  $l_1$  and intersects  $l_1$  at the point  $B$ .

(a) Find the equation of the line  $l_1$  in the form  $ax + by + c = 0$  where  $a$ ,  $b$  and  $c$  are integers .....

.....

.....

(b) Find the coordinates of the point where  $l_2$  intersects the  $x$ -axis .....

.....

.....

8. The lines  $x = 2$  and  $x = 7$  are tangent to a circle and  $y = 4$  touches the top of the circle. Find the equation of the circle in the form  $(ax + b)^2 + (ay + c)^2 = d$ , where  $a$ ,  $b$ ,  $c$  and  $d$  are integers. ....

.....

.....

.....

.....

9. If  $x$  is so small that  $x$  to the power of 3 or higher can be ignored, show that

$$(3 - x)(1 + 2x)^4 \approx 3 + 23x + 64x^2$$

.....

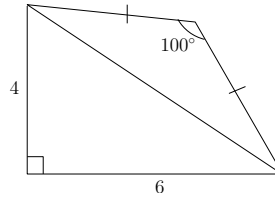
.....

.....

.....

.....

10. Calculate the area of this quadrilateral:



.....  
 .....  
 .....  
 .....

11. Triangle  $PQR$  is such that  $PQ = 3\text{cm}$ ,  $PR = 4\text{cm}$  and angle  $QPR = \frac{2}{5}\pi$ . An arc of a circle, centre at  $P$  and radius  $3\text{cm}$  starts at  $Q$  and cuts  $PR$  at  $S$ . Find the perimeter and area of the region enclosed by the arc  $QS$  and the sides  $SR$  and  $QR$ .....

.....  
 .....  
 .....  
 .....

12. Without the use of a calculator, evaluate the following:

(a)  $\cos(270)$  .....

(b)  $\sin\left(\frac{-\pi}{2}\right)$ .....

(c)  $\tan(180)$ .....

13. Solve  $\cos(4t) = \frac{2}{3}$  on the interval  $0 \leq t \leq 360$ .....

.....  
 .....  
 .....  
 .....

14. Solve  $2\log_4(x) - \log_4(2x - 3) = 1$  for  $x$ .....

.....  
 .....  
 .....

15. Find  $y$  given that  $\frac{dy}{dx} = \frac{1}{\sqrt{x}}$  and  $y$  passes through the point  $(9, 9)$ .....  
 .....  
 .....  
 .....  
 .....
16. Find and classify the stationary points of  $y = 2x^3 - 3x^2 - 36x + 14$ .....  
 .....  
 .....  
 .....
17. Find the equation of the tangent to the curve of  $y = x^2(1 - x)$  at the point where  $x = 1$ .....  
 .....  
 .....  
 .....
18. (a) Find the  $x$ -coordinates of where the graphs of  $y = 3x - x^2$  and  $y = 2x$  intersect.  
 .....  
 .....  
 .....
- (b) Hence, find the area of the region enclosed by the two graphs.....  
 .....  
 .....  
 .....