Cubics - Past Edexcel Exam Questions

1. **(Question 9 - May 2018)**

The curve C has equation $y = f(x)$, where

$$f'(x) = (x - 3)(3x + 5)$$

Given that the point $P(1, 20)$ lies on C,

(a) find $f(x)$, simplifying each term. [5]

(b) Show that

$$f(x) = (x - 3)^2(x + A)$$

where A is a constant to be found. [3]

(c) Sketch the graph of C. Show clearly the coordinates of the points where C cuts or meets the x-axis and where C cuts the y-axis. [4]

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2. **(Question 8 - May 2015)**

(a) Factorise completely $9x - 4x^3$. [3]

(b) Sketch the curve C with equation

$$y = 9x - 4x^3.$$

Show on your sketch the coordinates at which the curve meets the x-axis. [3]

The points A and B lie on C and have x coordinates of -2 and 1 respectively.

(c) Show that the length of AB is $k\sqrt{10}$ where k is a constant to be found. [4]

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3. **(Question 4 - May 2014)**

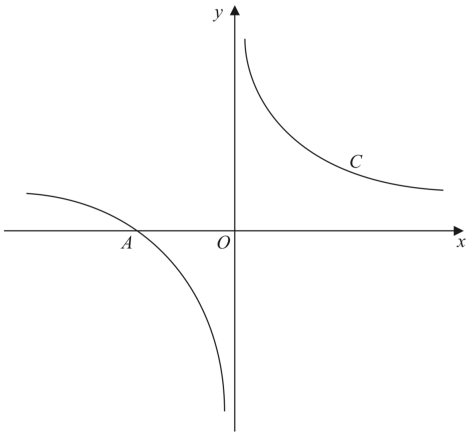

Figure 1

Figure 1 shows a sketch of the curve C with equation

$$y = \frac{1}{x} + 1, \quad x \neq 0.$$

The curve C crosses the x-axis at the point A.

(a) State the x-coordinate of the point A. [1]

The curve D has equation $y = x^2(x - 2)$ for all real values of x.

(b) A copy of Figure 1 is shown below.
On this copy, sketch the graph of curve D.
Show on the sketch the coordinates of each point where the curve D crosses the coordinate axes. [3]

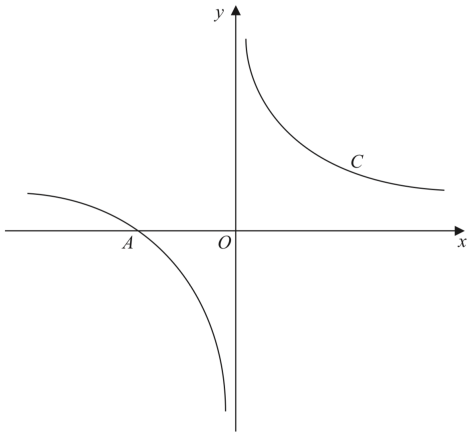

Figure 1

(c) Using your sketch, state, giving a reason, the number of real solutions to the equation

$$x^2(x-2) = \frac{1}{x} + 1.$$

[1]

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4. **(Question 1 - Jan 2013)**

Factorise completely $x - 4x^3$. [3]

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5. **(Question 8b+d - Jan 2012)**

The curve C_1 has equation

$$y = x^2(x+2).$$

(a) (*Differentiation question*)

(b) Sketch C_1, showing the coordinates of the points where C_1 meets the x-axis. [3]

(c) (*Differentiation question*)

The curve C_2 has equation

$$y = (x-k)^2(x-k+2)$$

where k is a constant and $k > 2$.

(d) Sketch C_2, showing the coordinates of the points where C_2 meets the x and y axes. [3]

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6. **(Question 10a - May 2011)**

The curve C has equation

$$y = (x+1)(x+3)^2.$$

(a) Sketch C, showing the coordinates of the points at which C meets the axes. [4]

b), c) and d) are differentiation and coordinate geometry questions.

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7. **(Question 10 - Jan 2011)**

(a) Sketch the graphs of

i. $y = x(x+2)(3-x)$,

ii. $y = -\frac{2}{x}$,

showing clearly the coordinates of all the points where the curves cross the coordinate axes. [6]

(b) Using your sketch state, giving a reason, the number of real solutions to the equation

$$x(x+2)(3-x) + \frac{2}{x} = 0$$

[2]

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8. **(Question 10 - May 2010)**

(a) Sketch the graphs of

i. $y = x(4-x)$,

ii. $y = x^2(7-x)$,

showing clearly the coordinates of the points where the curves cross the coordinate axes. [5]

(b) Show that the x-coordinates of the points of intersection of

$$y = x(4-x) \qquad \text{and} \qquad y = x^2(7-x)$$

are given by the solutions to the equation $x\left(x^2 - 8x + 4\right) = 0$. [3]

The point A lies on both the curves and the x and y coordinates of A are both positive.

(c) Find the exact coordinates of A, leaving your answer in the form $\left(p + q\sqrt{3}, r + s\sqrt{3}\right)$, where p, q, r and s are integers. [7]

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9. **(Question 9 - Jan 2010)**

(a) Factorise completely $x^3 - 4x$. [3]

(b) Sketch the curve with equation

$$y = x^3 - 4x,$$

showing the coordinates of the points at which the curve meets the x-axis. [3]

The point A with x-coordinate -1 and the point B with x-coordinate 3 lie on the curve C.

(c) Find an equation of the line which passes through A and B, giving your answer in the form $y = mx + c$, where m and c are constants. [5]

(d) Show that the length of AB is $k\sqrt{10}$, where k is a constant to be found. [2]

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10. **(Question 10 - Jun 2009)**

(a) Factorise completely $x^3 - 6x^2 + 9x$. [3]

(b) Sketch the curve with equation

$$y = x^3 - 6x^2 + 9x$$

showing the coordinates of the points at which the curve meets the x-axis. [4]

Using your answer to part (b), or otherwise,

(c) sketch, on a separate diagram, the curve with equation

$$y = (x-2)^3 - 6(x-2)^2 + 9(x-2)$$

showing the coordinates of the points at which the curve meets the x-axis. [2]

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11. **(Question 8 - Jan 2009)**

The point $P(1, a)$ lies on the curve with equation $y = (x+1)^2(2-x)$.

(a) Find the value of a. [1]

(b) Sketch the curves with the following equations:

i. $y = (x+1)^2(2-x)$,

ii. $y = \frac{2}{x}$.

On your diagram show clearly the coordinates of any points at which the curves meet the axes. [5]

(c) With reference to your diagram in part (b), state the number of real solutions to the equation

$$(x+1)^2(2-x) = \frac{2}{x}.$$

[1]

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12. **(Question 2 - Jun 2008)**

Factorise completely

$$x^3 - 9x.$$

[3]

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13. **(Question 10 - Jan 2008)**

The curve C has equation

$$y = (x+3)(x-1)^2.$$

(a) Sketch C, showing clearly the coordinates of the points where the curve meets the coordinate axes. [4]

(b) Show that the equation of C can be written in the form

$$y = x^3 + x^2 - 5x + k,$$

where k is a positive integer, and state the value of k. [2]

There are two points on C where the gradient of the tangent to C is equal to 3.

(c) Find the x-coordinates of these two points. [6]

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14. **(Question 10 - Jan 2007)**

(a) On the same axes sketch the graphs of the curves with equations

i. $y = x^2(x - 2)$ [3]

ii. $y = x(6 - x)$ [3]

and indicate on your sketches the coordinates of all the points where the curves cross the x-axis.

(b) Use algebra to find the coordinates of the points where the graphs intersect. [7]

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15. **(Question 9 - May 2006)**

Given that $f(x) = (x^2 - 6x)(x - 2) + 3x$,

(a) express $f(x)$ in the form $x(ax^2 + bx + c)$, where a, b and c are constants. [3]

(b) Hence factorise $f(x)$ completely. [2]

(c) Sketch the graph of $y = f(x)$, showing the coordinates of each point at which the graph meets the axes. [3]

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16. **(Question 1 - Jan 2006)**

Factorise completely

$$x^3 - 4x^2 + 3x.$$

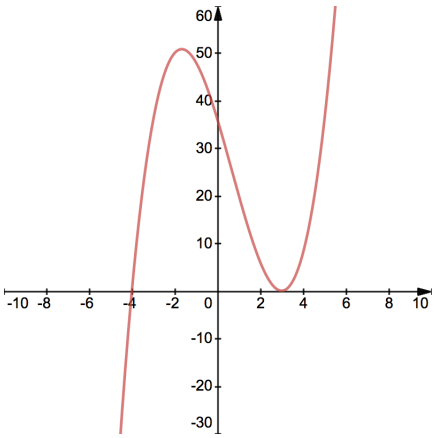
[3]

Solutions

1. (a) $f(x) = x^3 - 2x^2 - 15x + 36$ (*Differentiation Question*)

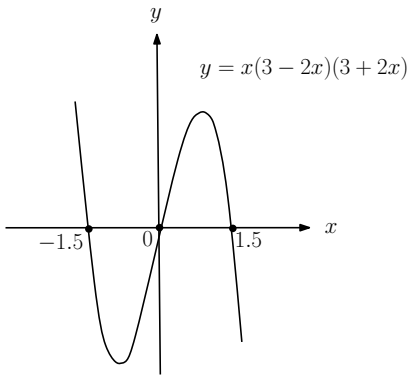
   (b) $f(x) = (x-3)^2(x+4)$, $A = 4$

   (c) See figure below.



2. (a) $x(3-2x)(3+2x)$

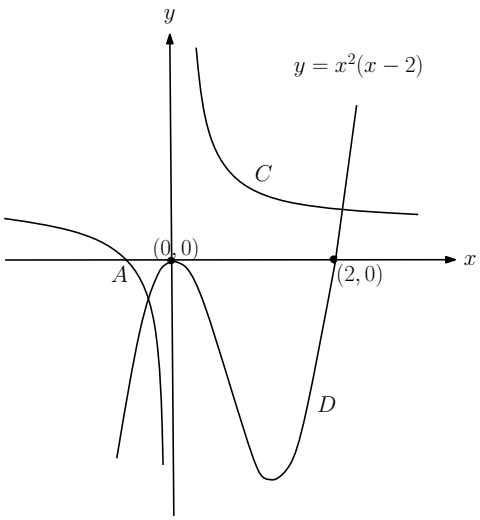
   (b) See figure below.



   (c) $k = 3$

3. (a) $x = -1$

   (b) See figure below.

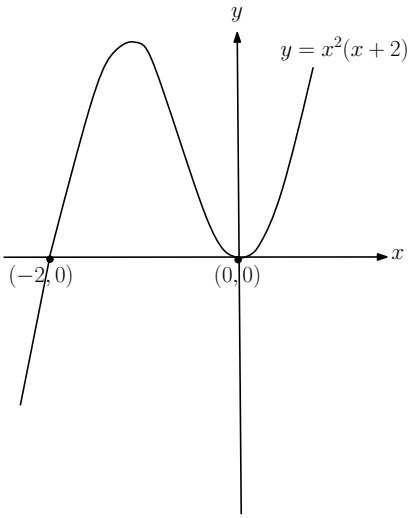


(c) 2 real solutions as we can see from the graph that the curves intersect twice.

4. $x(1-2x)(1+2x)$

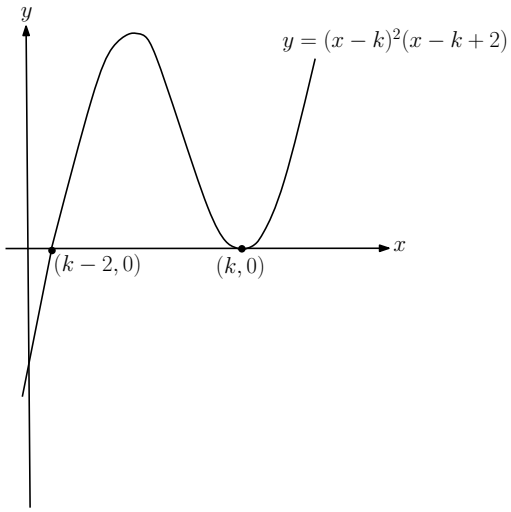
5. (a) (*Differentiation*)

   (b) See figure below.

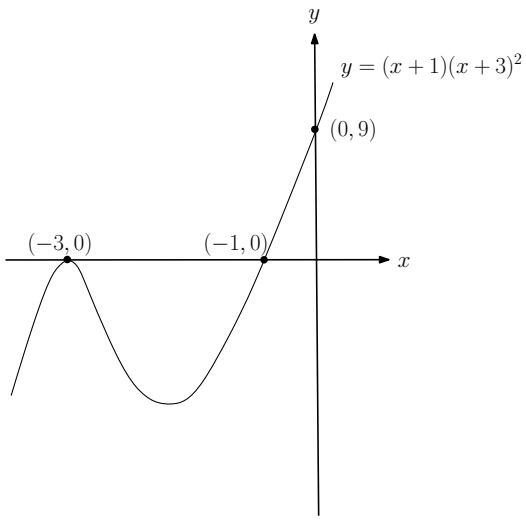


(c) (*Differentiation*)

(d) See figure below.

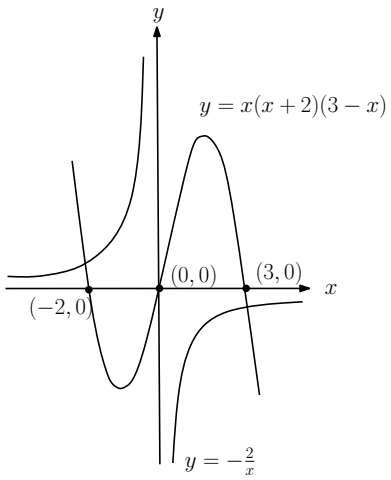


6. (a) See figure below.



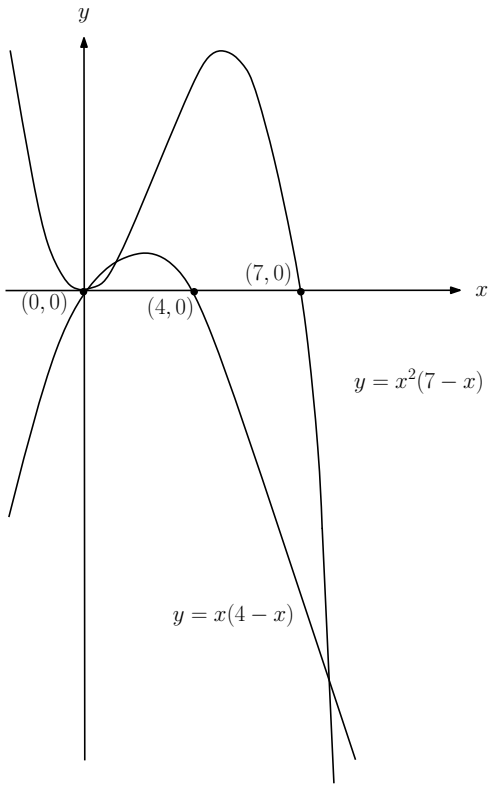
b), c) and d) are differentiation and coordinate geometry questions

7. (a) See figure below.



(b) There are 2 solutions since the curves intersect twice.

8. (a) See figure below.

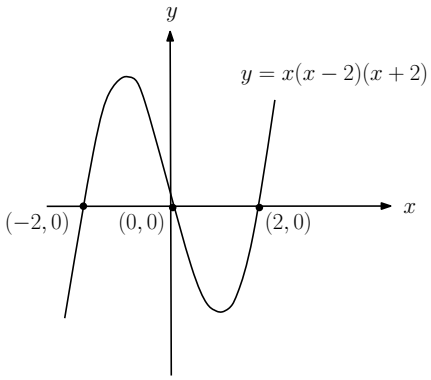


(b) -

(c) $(4-2\sqrt{3}, 8\sqrt{3}-12)$. Note that both $4+2\sqrt{3}$ and $4-2\sqrt{3}$ are both positive but only the y-coordinate of $8\sqrt{3}-12$ is positive.

9. (a) $x(x-2)(x+2)$

(b) See figure below.

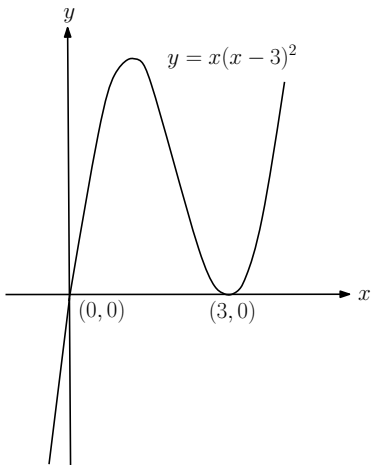


(c) $y = 3x + 6$

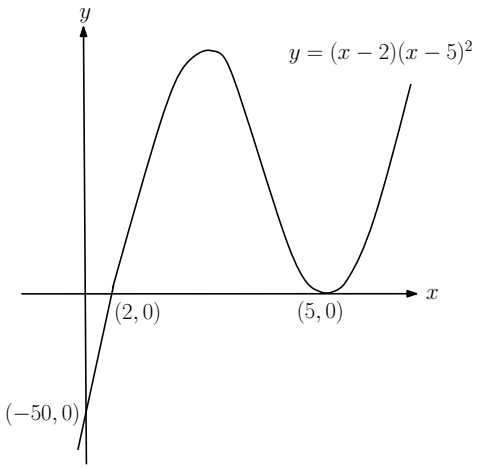
(d) $k = 4$

10. (a) $x(x-3)^2$

(b) See figure below.

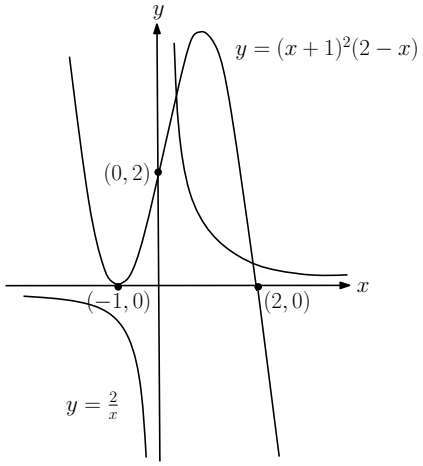


(c) See figure below.



11. (a) $a = 4$

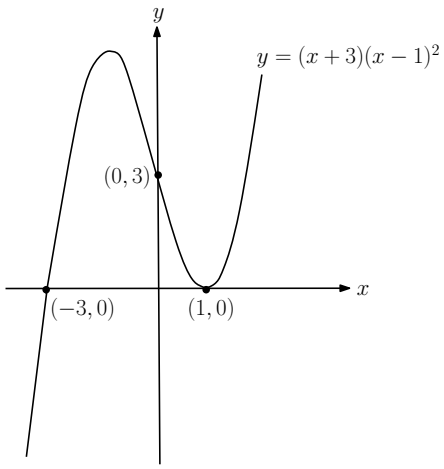
    (b) See figure below.



    (c) The graphs intersect twice and so there are 2 solutions. We know they intersect twice since the point (1,4), on the reciprocal functions, lies above the point (1,2) on the cubic.

12. $x(x-3)(x+3)$

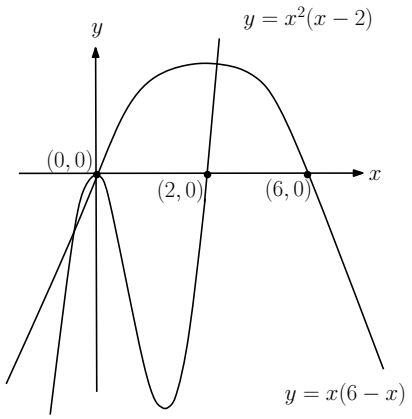
13. (a) See figure below.



(b) $k = 3$

(c) $x = \frac{4}{3}$, $x = -2$ (*Differentiation*)

14. (a) See figure below.

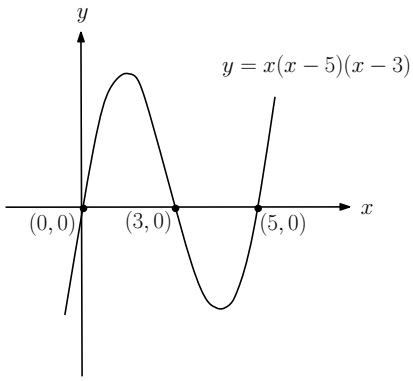


(b) $(-2, -16)$, (3,9)

15. (a) $x\left(x^2 - 8x + 15\right)$

(b) $x(x-5)(x-3)$

(c) See figure below.



16. $x(x-3)(x-1)$