Differentiation - Past Edexcel Exam Questions

1. **(Question 2 - C1 May 2018)**

Given

$$y = 3\sqrt{x} - 6x + 4, \qquad x > 0$$

(a) *(Integration Question)*

(b) i. Find $\frac{dy}{dx}$.

ii. Hence find the value of x such that $\frac{dy}{dx} = 0$.

[4]

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2. **(Question 10 - C1 May 2018)**

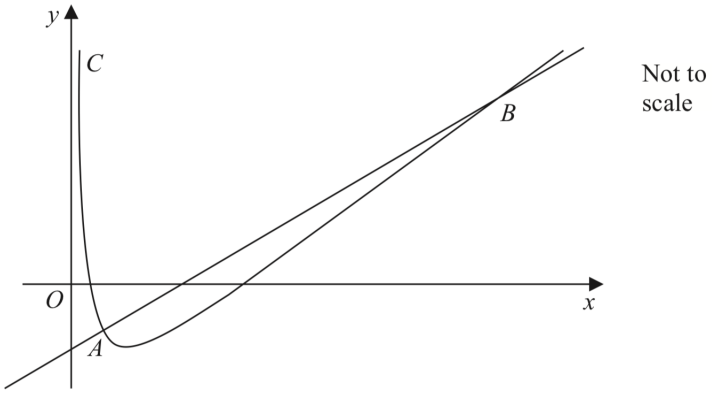

Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = \frac{1}{2}x + \frac{27}{x} - 12, \qquad x > 0$$

The point A lies on C and has coordinates $\left(3, -\frac{3}{2}\right)$.

(a) Show that the equation of the normal to C at the point A can be written as $10y = 4x - 27$. [5]

(b) *(Simultaneous Equations Question)*

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3. **(Question 2 - C1 May 2017)**

Given

$$y = \sqrt{x} + \frac{4}{\sqrt{x}} + 4, \qquad x > 0$$

find the value of $\frac{dy}{dx}$ when $x = 8$, writing your answer in the form $a\sqrt{2}$, where a is a rational number. [5]

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4. **(Question 7 - C1 May 2017)**

The curve C has equation $y = f(x)$, $x > 0$, where

$$f'(x) = 30 + \frac{6 - 5x^2}{\sqrt{x}}.$$

Given that the point $(4, -8)$ lies on C,

(a) find the equation of the tangent to C at P, giving your answer in the form $y = mx + c$, where m and c are constants. [4]

(b) *(Integration Question)*

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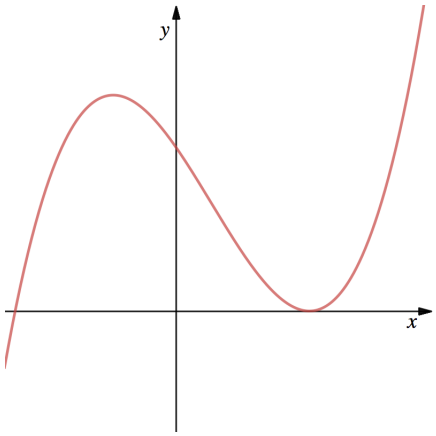
5. **(Question 10 - C1 May 2017)**



This figure shows a sketch of part of the curve $y = f(x)$, $x \in \mathbb{R}$, where

$$f(x) = (2x - 5)^2(x + 3).$$

(a) *(Transformations Question)*

(b) Show that $f'(x) = 12x^2 - 16x - 35$. [3]

Points A and B are distinct points that lie on the curve $y = f(x)$.
The gradient of the curve at A is equal to the gradient of the curve at B.
Given that point A has x-coordinate 3,

(c) find the x-coordinate of point B. [5]

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6. **(Question 7 - C1 May 2016)**

Given that

$$y = 3x^2 + 6x^{\frac{1}{3}} + \frac{2x^3 - 7}{3\sqrt{x}}, \qquad x > 0$$

find $\frac{dy}{dx}$. Give each term in your answer in its simplest form. [6]

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7. **(Question 11 - C1 May 2016)**

The curve C has equation $y = 2x^3 + kx^2 + 5x + 6$, where k is a constant.

(a) Find $\frac{dy}{dx}$. [2]

The point P, where $x = -2$, lies on C.
The tangent to C at the point P is parallel to the line with equation $2y - 17x - 1 = 0$.
Find

(b) the value of k, [4]

(c) the value of the y-coordinate of P, [2]

(d) the equation of the tangent to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [2]

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8. **(Question 3 - C1 May 2015)**

Given that $y = 4x^3 - \frac{5}{x^2}$, $x \neq 0$, find in their simplest form

(a) $\frac{dy}{dx}$ [3]

(b) *(Integration Question)*

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9. **(Question 6 - C1 May 2015)**

The curve C has equation

$$y = \frac{(x^2 + 4)(x - 3)}{2x}, \qquad x \neq 0$$

(a) Find $\frac{dy}{dx}$ in its simplest form. [5]

(b) Find an equation of the tangent to C at the point where $x = -1$.
Give your answer in the form $ax + by + c = 0$ where a, b and c are integers. [5]

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10. **(Question 10 - C1 May 2015)**

A curve with equation $y = f(x)$ passes through the point $(4, 9)$.

Given that

$$f'(x) = \frac{3\sqrt{x}}{2} - \frac{9}{4\sqrt{x}} + 2, \quad x > 0$$

(a) *(Integration Question)*

Point P lies on the curve.

The normal to the curve at P is parallel to the line $2y + x = 0$.

(b) Find the x-coordinate of P. [5]

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11. **(Question 7 - C1 May 2014)**

Differentiate with respect to x, giving each answer in its simplest form.

(a) $(1 - 2x)^2$ [3]

(b) $\frac{x^5+6\sqrt{x}}{2x^2}$ [4]

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12. **(Question 6 - C1 May 2015)**

A curve with equation $y = f(x)$ passes through the point $(4, 25)$.

Given that

$$f'(x) = \frac{3}{8}x^2 - 10x^{-\frac{1}{2}} + 1, \quad x > 0$$

(a) *(Integration Question)*

(b) Find an equation of the normal to the curve at the point $(4, 25)$.

Give your answer in the form $ax + by + c = 0$, where A, b and c are integers to be found. [5]

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13. **(Question 9 - C1 May 2013)**

$$f'(x) = \frac{(3-x^2)^2}{x^2}, \quad x \neq 0$$

(a) Show that

$$f'(x) = 9x^{-2} + A + Bx^2,$$

where A and B are constants to be found. [3]

(b) Find $f''(x)$. [2]

(c) *(Integration Question)*

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14. **(Question 11 - C1 May 2013)**

11.

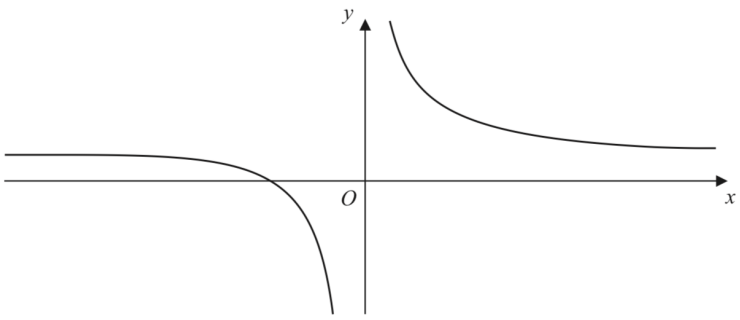

Figure 2

Figure 2 shows a sketch of the curve H with equation $y = \frac{3}{x} + 4, \ x \neq 0$.

(a) Give the coordinates of the point where H crosses the x-axis.

(1)

(b) Give the equations of the asymptotes to H.

(2)

(c) Find an equation for the normal to H at the point $P(-3, 3)$.

(5)

This normal crosses the x-axis at A and the y-axis at B.

(d) Find the length of the line segment AB. Give your answer as a surd.

(3)

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15. **(Question 11 - C1 January 2013)**

The curve C has equation

$$y = 2x - 8\sqrt{x} + 5, \quad x \geq 0$$

(a) Find $\frac{dy}{dx}$, giving each term in its simplest form. [3]

The point P on C has x-coordinate equal to $\frac{1}{4}$.

(b) Find the equation of the tangent to C at the point P, giving your answer in the form $y = ax + b$, where a and b are constants. [4]

The tangent to C at the point Q is parallel to the line with equation $2x - 3y + 18 = 0$.

(c) Find the coordinates of Q. [5]

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16. **(Question 4 - C1 May 2012)**

$$y = 5x^3 - 6x^{\frac{4}{3}} + 2x - 3.$$

(a) Find $\frac{dy}{dx}$, giving each term in its simplest form. [4]

(b) Find $\frac{d^2y}{dx^2}$. [2]

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17. **(Question 7 - C1 May 2012)**

The point $P(4, -1)$ lies on the curve C with equation $y = f(x)$, $x > 0$, and

$$f'(x) = \frac{1}{2}x - \frac{6}{\sqrt{x}} + 3$$

(a) Find the equation of the tangent to C at the point P, giving your answer in the form $y = mx + c$, where m and c are integers. [4]

(b) *(Integration Question)*

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18. **(Question 1 - C1 January 2012)**

Given that $y = x^4 + 6x^{\frac{1}{2}}$, find in their simplest form

(a) $\frac{dy}{dx}$, [3]

(b) *(Integration Question)*

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19. **(Question 8 - C1 January 2012)**

The curve C_1 has equation

$$y = x^2(x + 2)$$

(a) Find $\frac{dy}{dx}$. [2]

(b) Sketch C_1, showing the coordinates of the points where C_1 meets the x-axis. [3]

(c) Find the gradient of C_1 at each point where C_1 meets the x-axis. [2]

The curve C_2 has equation

$$y = (x - k)^2(x - k + 2)$$

where k is a constant and $k > 2$.

(d) Sketch C_2, showing the coordinates of the points where C_2 meets the x and y axes. [3]

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20. **(Question 10 - C1 January 2012)**

10.

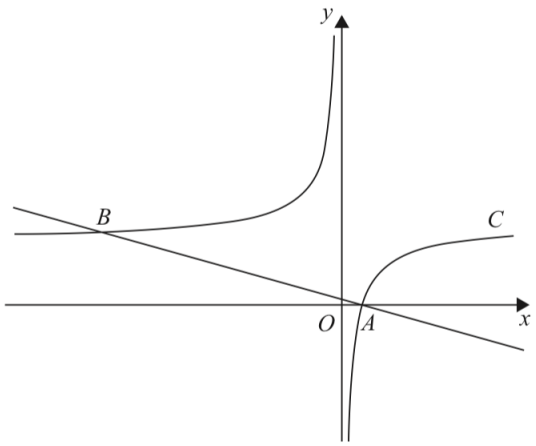

Figure 2

Figure 2 shows a sketch of the curve C with equation

$$y = 2 - \frac{1}{x}, \quad x \neq 0$$

The curve crosses the x-axis at the point A.

(a) Find the coordinates of A.

(1)

(b) Show that the equation of the normal to C at A can be written as

$$2x + 8y - 1 = 0$$

(6)

The normal to C at A meets C again at the point B, as shown in Figure 2.

(c) Find the coordinates of B.

(4)

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21. **(Question 2 - C1 May 2011)**

Given that $y = 2x^5 + 7 + \frac{1}{x^3}$, $x \neq 0$, find, in their simplest form,

(a) $\frac{dy}{dx}$, [3]

(b) *(Integration Question)*

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22. **(Question 10 - C1 May 2011)**

The curve C has equation

$$y = (x+1)(x+3)^2.$$

(a) Sketch C, showing the coordinates of the points at which C meets the axes. [4]

(b) Show that $\frac{dy}{dx} = 3x^2 + 14x + 15$. [3]

The point A, with x-coordinate -5, lies on C.

(c) Find the equation of the tangent to C at A, giving your answer in the form $y = mx + c$, where m and c are constants. [4]

Another point B also lies on C. The tangents to C at A and B are parallel.

(d) Find the x-coordinate of B. [3]

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23. **(Question 11 - C1 January 2011)**

The curve C has equation

$$y = \frac{1}{2}x^3 - 9x^{\frac{3}{2}} + \frac{8}{x} + 30, \quad x > 0.$$

(a) Find $\frac{dy}{dx}$. [4]

(b) Show that the point $P(4, -8)$ lies on C. [2]

(c) Find an equation of the normal to C at the point P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [6]

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24. **(Question 7 - C1 May 2010)**

Given that

$$y = 8x^3 - 4\sqrt{x} + \frac{3x^2 + 2}{x}, \quad x > 0,$$

find $\frac{dy}{dx}$. [6]

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25. **(Question 11 - C1 May 2010)**

11. The curve C has equation $y = f(x), \quad x > 0$, where

$$\frac{dy}{dx} = 3x - \frac{5}{\sqrt{x}} - 2$$

Given that the point $P(4, 5)$ lies on C, find

(a) $f(x)$,

(5)

(b) an equation of the tangent to C at the point P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

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26. **(Question 1 - C1 January 2010)**

Given that $y = x^4 + x^{\frac{1}{3}} + 3$, find $\frac{dy}{dx}$. [3]

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27. **(Question 6 - C1 January 2010)**

The curve C has equation

$$y = \frac{(x+3)(x-8)}{x}, \quad x > 0.$$

(a) Find $\frac{dy}{dx}$ in its simplest form. [4]

(b) Find an equation of the tangent to C at the point where $x = 2$. [4]

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28. **(Question 3 - C1 June 2009)**

Given that $y = 2x^3 + \frac{3}{x^2}$, $x \neq 0$, find

(a) $\frac{dy}{dx}$, [3]

(b) *(Integration Question)*

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29. **(Question 9 - C1 June 2009)**

$$f(x) = \frac{(3-4\sqrt{x})^2}{\sqrt{x}}, \quad x > 0$$

(a) Show that $f(x) = 9x^{-\frac{1}{2}} + Ax^{\frac{1}{2}} + B$, where A and B are constants to be found. [3]

(b) Find $f'(x)$. [3]

(c) Evaluate $f'(9)$. [2]

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30. **(Question 11 - C1 June 2009)**

The curve C has equation

$$y = x^3 - 2x^2 - x + 9, \quad x > 0$$

The point P has coordinates (2,7).

(a) Show that P lies on C. [1]

(b) Find the equation of the tangent to C at P, giving your answer in the form $y = mx + c$, where m and c are constants. [5]

The point Q also lies on C.

Given that the tangent to C at Q is perpendicular to the tangent to C at P,

(c) show that the x-coordinate of Q is $\frac{1}{3}(2+\sqrt{6})$. [5]

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31. **(Question 6 - C1 January 2009)**

Given that $\frac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$ can be written in the form $2x^p - x^q$,

(a) write down the value of p and the value of q. [2]

Given that $y = 5x^4 - 3 + \frac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$, $x > 0$,

(b) find $\frac{dy}{dx}$, simplifying the coefficient of each term. [4]

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32. **(Question 11 - C1 January 2009)**

The curve C has equation

$$y = 9 - 4x - \frac{8}{x}, \quad x > 0.$$

The point P on C has x-coordinate equal to 2.

(a) Show that the equation of the tangent to C at the point P is $y = 1 - 2x$. [6]

(b) Find an equation of the normal to C at the point P. [3]

The tangent at P meets the x-axis at A and the normal at P meets the x-axis at B.

(c) Find the area of the triangle APB. [4]

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33. **(Question 4 - C1 June 2008)**

$$f(x) = 3x + x^3, \quad x > 0.$$

(a) Differentiate to find $f'(x)$. [2]

Given that $f'(x) = 15$,

(b) find the value of x. [3]

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34. **(Question 9 - C1 June 2008)**

The curve C has equation $y = kx^3 - x^2 + x - 5$, where k is a constant.

(a) Find $\frac{dy}{dx}$. [2]

The point A with x-coordinate $-\frac{1}{2}$ lies on C. The tangent to C at A is parallel to the line with equation $2y - 7x + 1 = 0$.

Find

(b) the value of k, [4]

(c) the value of the y-coordinate of A. [2]

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35. **(Question 5 - C1 January 2008)**

(a) Write $\frac{2\sqrt{x}+3}{x}$ in the form $2x^p + 3x^q$, where p and q are constants. [2]

Given that $y = 5x - 7 + \frac{2\sqrt{x}+3}{x}$, $x > 0$,

(b) find $\frac{dy}{dx}$, simplifying the coefficient of each term. [4]

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36. **(Question 9 - C1 January 2008)**

9. The curve C has equation $y = f(x)$, $x > 0$, and $f'(x) = 4x - 6\sqrt{x} + \frac{8}{x^2}$.

Given that the point $P(4, 1)$ lies on C,

(a) find $f(x)$ and simplify your answer.

(6)

(b) Find an equation of the normal to C at the point $P(4, 1)$.

(4)

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37. **(Question 10 - C1 January 2008)**

10. The curve C has equation

$$y = (x+3)(x-1)^2 .$$

(a) Sketch C showing clearly the coordinates of the points where the curve meets the coordinate axes.

(4)

(b) Show that the equation of C can be written in the form

$$y = x^3 + x^2 - 5x + k,$$

where k is a positive integer, and state the value of k.

(2)

There are two points on C where the gradient of the tangent to C is equal to 3.

(c) Find the x-coordinates of these two points.

(6)

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38. **(Question 3 - C1 May 2007)**

Given that $y = 3x^2 + 4\sqrt{x}$, $x > 0$, find

(a) $\frac{dy}{dx}$, [2]

(b) $\frac{d^2y}{dx^2}$. [2]

(c) *(Integration Question)*

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39. **(Question 10 - C1 May 2007)**

The curve C has equation $y = x^2(x-6) + \frac{4}{x}$, $x > 0$.

The points P and Q lie on C and have x-coordinates 1 and 2 respectively.

(a) Show that the length of PQ is $\sqrt{170}$. [4]

(b) Show that the tangents to C at P and Q are parallel. [5]

(c) Find an equation for the normal to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [4]

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40. **(Question 1 - C1 January 2007)**

Given that

$$y = 4x^3 - 1 + 2x^{\frac{1}{2}}, \quad x > 0,$$

find $\frac{dy}{dx}$. [4]

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41. **(Question 7 - C1 January 2007)**

The curve C has equation $y = f(x)$, $x \neq 0$, and the point $P(2, 1)$ lies on C. Given that

$$f'(x) = 3x^2 - 6 - \frac{8}{x^2}$$

(a) *(Integration Question).*

(b) Find an equation for the tangent to C at the point P, giving your answer in the form $y = mx + c$, where m and c are integers. [4]

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42. **(Question 8 - C1 January 2007)**

The curve C has equation $y = 4x + 3x^{\frac{3}{2}} - 2x^2$, $x > 0$.

(a) Find an expression for $\frac{dy}{dx}$. [3]

(b) Show that the point $P(4, 8)$ lies on C. [1]

(c) Show that an equation of the normal to C at the point P is

$$3y = x + 20.$$

[4]

The normal to C at P cuts the x-axis at the point Q.

(d) Find the length PQ, giving your answer in a simplified surd form. [3]

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43. **(Question 5 - C1 May 2006)**

Differentiate with respect to x

(a) $x^4 + 6\sqrt{x}$, [3]

(b) $\frac{(x+4)^2}{x}$. [4]

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44. **(Question 10 - C1 May 2006)**

10. The curve C with equation $y = f(x)$, $x \neq 0$, passes through the point $(3, 7\frac{1}{2})$.

Given that $f'(x) = 2x + \frac{3}{x^2}$,

(a) find $f(x)$.

(5)

(b) Verify that $f(-2) = 5$.

(1)

(c) Find an equation for the tangent to C at the point $(-2, 5)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

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45. **(Question 4 - C1 January 2006)**

Given that $y = 2x^2 - \frac{6}{x^3}$, $x \neq 0$,

(a) find $\frac{dy}{dx}$, [2]

(b) *(Integration Question)*

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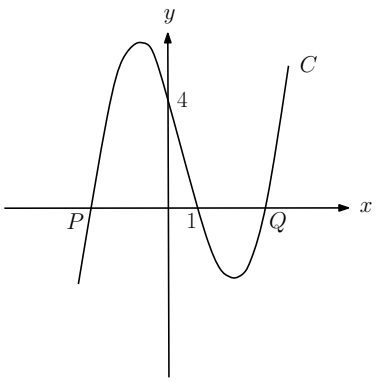
46. **(Question 9 - C1 January 2006)**

This figure shows part of the curve C with equation $y = (x - 1)(x^2 - 4)$.



The curve cuts the x-axis at the points P, (1,0) and Q.

(a) Write down the x-coordinate of P and the x-coordinate of Q. [2]

(b) Show that $\frac{dy}{dx} = 3x^2 - 2x - 4$. [3]

(c) Show that $y = x + 7$ is an equation of the tangent to C at the point $(-1, 6)$. [2]

The tangent to C at the point R is parallel to the tangent at the point $(-1, 6)$.

(d) Find the exact coordinates of R. [5]

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47. **(Question 2 - C1 May 2005)**

Given that $y = 6x - \frac{4}{x^2}$, $x \neq 0$,

(a) find $\frac{dy}{dx}$. [2]

(b) *(Integration Question)*

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48. **(Question 10 - C1 May 2005)**

The curve C has equation $y = \frac{1}{3}x^3 - 4x^2 + 8x + 3$.

The point P has coordinates $(3, 0)$.

(a) Show that P lies on C. [1]

(b) Find the equation of the tangent to C at P, giving your answer in the form $y = mx + c$, where m and c are constants. [5]

Another point Q also lies on C. The tangent to C at Q is parallel to the tangent to C at P.

(c) Find the coordinates of Q. [5]

Solutions

1. (a) *(Integration Question)*
   (b) i. $\frac{3}{2}x^{-\frac{1}{2}} - 6$
   ii. $x = \frac{1}{16}$
2. (a) -
   (b) *(Simultaneous Equations Question)*
3. $\frac{1}{16}\sqrt{2}$
4. (a) $y = -7x + 20$
   (b) *(Integration Question)*
5. (a) *(Transformations Question)*
   (b) -
   (c) $x = \frac{5}{3}$
6. $6x + 2x^{-\frac{2}{3}} + \frac{5}{3}x^{\frac{3}{2}} + \frac{7}{6}x^{-\frac{3}{2}}$
7. (a) $6x^2 + 2kx + 5$
   (b) $k = \frac{41}{8}$
   (c) $y = \frac{1}{2}$
   (d) $-17x + 2y - 35 = 0$
8. (a) $12x^2 + \frac{10}{x^3}$
   (b) *(Integration Question)*
9. (a) $x - \frac{3}{2} + \frac{6}{x^2}$
   (b) $7x - 2y + 27 = 0$
10. (a) *(Integration Question)*
    (b) $x = 1.5$
11. (a) $-4 + 8x$
    (b) $\frac{3}{2}x^2 - \frac{9}{2}x^{-\frac{5}{2}}$
12. (a) *(Integration Question)*

(b) $x + 2y - 54 = 0$, $a = 1$, $b = 2$ and $c = -54$

13. (a) $A = -6$, $B = 1$

    (b) $-18x^{-3} + 2x$

    (c) *(Integration Question)*

14. (a) $\left(-\frac{3}{4}, 0\right)$

    (b) $x = 0$, $y = 4$

    (c) $y = 3x + 12$

    (d) $4\sqrt{10}$

15. (a) $2 - 4x^{-\frac{1}{2}}$

    (b) $y = -6x + 3$

    (c) $(9, -1)$

16. (a) $15x^2 - 8x^{\frac{1}{3}} + 2$

    (b) $30x - \frac{8}{3}x^{-\frac{2}{3}}$

17. (a) $y = 2x - 9$

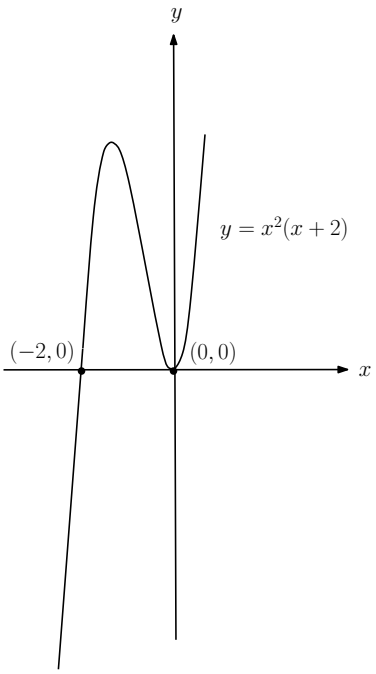
    (b) *(Integration Question)*

18. (a) $4x^3 + 3x^{-\frac{1}{2}}$

    (b) *(Integration Question)*

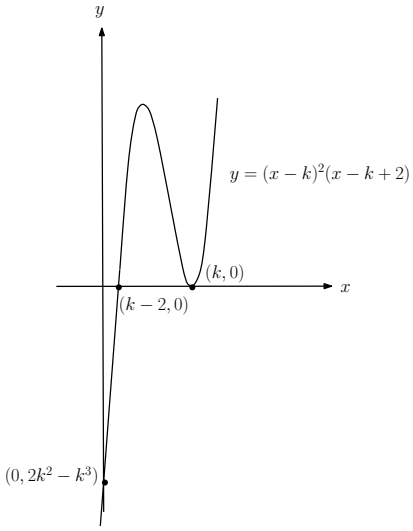
19. (a) $3x^2 + 4x$

    (b) See figure below.



(c) (4,0)

(d) See figure below.



20. (a) $\left(\frac{1}{2}, 0\right)$

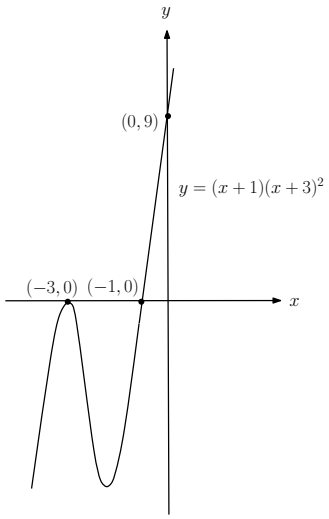
(b) -

(c) $\left(-8, \frac{17}{8}\right)$

21. (a) $10x^4 - 3x^{-4}$

(b) *(Integration Question)*

22. (a) .



(b) -

(c) $y = 20x + 84$

(d) $x = \frac{1}{3}$

23. (a) $\frac{3}{2}x^2 - \frac{27}{2}x^{\frac{1}{2}} - 8x^{-2}$

(b) -

(c) $2x - 7y - 64 = 0$

24. $24x^2 - 2x^{-\frac{1}{2}} + 3 - 2x^{-2}$

25. (a) *(Integration Question)*

(b) $15x - 2y - 50 = 0$

26. $4x^3 + \frac{1}{3}x^{-\frac{2}{3}}$

27. (a) $1 + 24x^{-2}$

(b) $y = 7x - 29$

28. (a) $6x^2 - 6x^{-3}$

(b) *(Integration Question)*

29. (a) $A = 16,\ B = -24$

(b) $-\frac{9}{2}x^{-\frac{3}{2}} + 8x^{-\frac{1}{2}}$

(c) $\frac{5}{2}$

30. (a) -
    (b) $y = 3x + 1$
    (c) -

31. (a) $p = \frac{3}{2}$, $q = 1$
    (b) $20x^3 + 3x^{\frac{1}{2}} - 1$

32. (a) -
    (b) $y = \frac{1}{2}x - 4$
    (c) $\frac{45}{4}$ units^2

33. (a) $3 + 3x^2$
    (b) $x = 2$

34. (a) $\frac{dy}{dx} = 3kx^2 - 2x + 1$
    (b) $k = 2$
    (c) $y = -6$

35. (a) $2x^{-\frac{1}{2}} + 3x^{-1}$
    (b) $\frac{dy}{dx} = 5 - x^{-\frac{3}{2}} - 3x^{-2}$

36. (a) *(Integration Question)*
    (b) $y = -\frac{2}{9}x + \frac{17}{9}$

37. (a) *(Curve Sketching Question)*
    (b) -
    (c) $x = -2$, $x = \frac{4}{3}$

38. (a) $6x + 2x^{-\frac{1}{2}}$
    (b) $6 - x^{-\frac{3}{2}}$
    (c) *(Integration Question)*

39. (a) -
    (b) -
    (c) $x - 13y - 14 = 0$

40. $12x^2 + x^{-\frac{1}{2}}$

41. (a) *(Integration Question)*
    (b) $y = 4x - 7$

42. (a) $4 + \frac{9}{2}x^{\frac{1}{2}} - 4x$
    (b) -
    (c) -
    (d) $8\sqrt{10}$

43. (a) $4x^3 + 3x^{-\frac{1}{2}}$
    (b) $1 - 16x^{-2}$

44. (a) *(Integration Question)*
    (b) -
    (c) $13x + 4y + 6 = 0$

45. (a) $4x + 18x^{-4}$
    (b) *(Integration Question)*

46. (a) $x_P = -2$, $x_Q = 2$
    (b) -
    (c) -
    (d) $\left(\frac{5}{3}, -\frac{22}{27}\right)$

47. (a) $6 + 8x^{-3}$
    (b) *(Integration Question)*

48. (a) -
    (b) $y = -7x + 21$
    (c) $\left(5, -\frac{46}{3}\right)$