Optimisation - Past Edexcel Exam Questions

(See 'Definite Integrals' for the 'Optimisation' exam questions from 2017 and 2018.)

1. **(Question 9 - C2 May 2016)**

9.

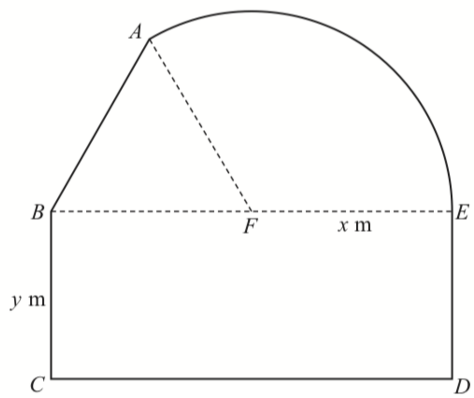

Figure 4

Figure 4 shows a plan view of a sheep enclosure.

The enclosure *ABCDEA*, as shown in Figure 4, consists of a rectangle *BCDE* joined to an equilateral triangle *BFA* and a sector *FEA* of a circle with radius x metres and centre *F*.

The points *B*, *F* and *E* lie on a straight line with $FE = x$ metres and $10 \leqslant x \leqslant 25$

(a) Find, in m^2, the exact area of the sector *FEA*, giving your answer in terms of x, in its simplest form. **(2)**

Given that $BC = y$ metres, where $y > 0$, and the area of the enclosure is $1000\ \text{m}^2$,

(b) show that

$$y = \frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right)$$

(3)

(c) Hence show that the perimeter P metres of the enclosure is given by

$$P = \frac{1000}{x} + \frac{x}{12}\left(4\pi + 36 - 3\sqrt{3}\right)$$

(3)

(d) Use calculus to find the minimum value of P, giving your answer to the nearest metre. **(5)**

(e) Justify, by further differentiation, that the value of P you have found is a minimum. **(2)**

2. **(Question 9 - C2 May 2015)**

9. A solid glass cylinder, which is used in an expensive laser amplifier, has a volume of 75π cm^3.
The cost of polishing the surface area of this glass cylinder is £2 per cm^2 for the curved surface area and £3 per cm^2 for the circular top and base areas.

Given that the radius of the cylinder is r cm,

(a) show that the cost of the polishing, £C, is given by

$$C = 6\pi r^2 + \frac{300\pi}{r}$$

(4)

(b) Use calculus to find the minimum cost of the polishing, giving your answer to the nearest pound.

(5)

(c) Justify that the answer that you have obtained in part (b) is a minimum.

(1)

3. **(Question 10 - C2 May 2014)**

10.

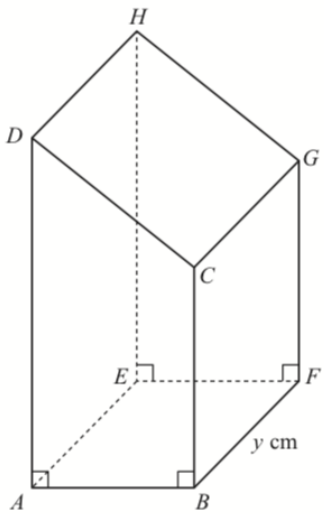

Figure 4

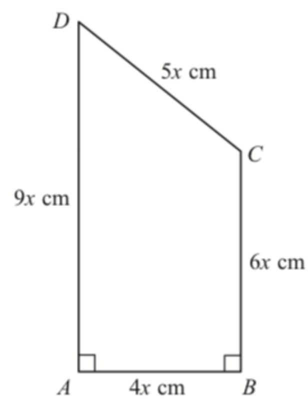

Figure 5

Figure 4 shows a closed letter box *ABFEHGCD*, which is made to be attached to a wall of a house.

The letter box is a right prism of length y cm as shown in Figure 4. The base *ABFE* of the prism is a rectangle. The total surface area of the six faces of the prism is $S\,\text{cm}^2$.

The cross section *ABCD* of the letter box is a trapezium with edges of lengths $DA = 9x$ cm, $AB = 4x$ cm, $BC = 6x$ cm and $CD = 5x$ cm as shown in Figure 5.
The angle $DAB = 90°$ and the angle $ABC = 90°$.

The volume of the letter box is 9600 cm^3.

(a) Show that

$$y = \frac{320}{x^2}$$

(2)

(b) Hence show that the surface area of the letter box, $S\,\text{cm}^2$, is given by

$$S = 60x^2 + \frac{7680}{x}$$

(4)

(c) Use calculus to find the minimum value of S.

(6)

(d) Justify, by further differentiation, that the value of S you have found is a minimum.

(2)

4. **(Question 9 - C2 May 2013)**

9. The curve with equation

$$y = x^2 - 32\sqrt{(x)} + 20, \qquad x > 0$$

has a stationary point P.

Use calculus

(a) to find the coordinates of P,

(6)

(b) to determine the nature of the stationary point P.

(3)

---

5. **(Question 8 - C2 January 2013)**

8. The curve C has equation $y = 6 - 3x - \dfrac{4}{x^3}$, $x \neq 0$

(a) Use calculus to show that the curve has a turning point P when $x = \sqrt{2}$

(4)

(b) Find the x-coordinate of the other turning point Q on the curve.

(1)

(c) Find $\dfrac{d^2y}{dx^2}$.

(1)

(d) Hence or otherwise, state with justification, the nature of each of these turning points P and Q.

(3)

---

6. **(Question 8 - C2 May 2012)**

8.

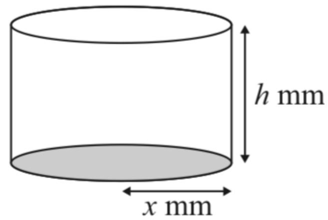

Figure 3

A manufacturer produces pain relieving tablets. Each tablet is in the shape of a solid circular cylinder with base radius x mm and height h mm, as shown in Figure 3.

Given that the volume of each tablet has to be 60 mm^3,

(a) express h in terms of x,

(1)

(b) show that the surface area, A mm^2, of a tablet is given by $A = 2\pi x^2 + \dfrac{120}{x}$

(3)

The manufacturer needs to minimise the surface area A mm^2, of a tablet.

(c) Use calculus to find the value of x for which A is a minimum.

(5)

(d) Calculate the minimum value of A, giving your answer to the nearest integer.

(2)

(e) Show that this value of A is a minimum.

(2)

7. **(Question 8 - C2 January 2012)**

8.

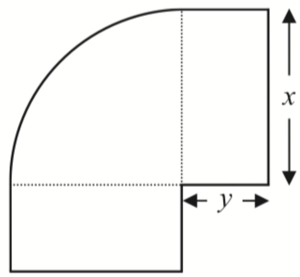

Figure 3

Figure 3 shows a flowerbed. Its shape is a quarter of a circle of radius x metres with two equal rectangles attached to it along its radii. Each rectangle has length equal to x metres and width equal to y metres.

Given that the area of the flowerbed is $4\ \text{m}^2$,

(a) show that

$$y = \frac{16 - \pi x^2}{8x}$$

(3)

(b) Hence show that the perimeter P metres of the flowerbed is given by the equation

$$P = \frac{8}{x} + 2x$$

(3)

(c) Use calculus to find the minimum value of P.

(5)

(d) Find the width of each rectangle when the perimeter is a minimum. Give your answer to the nearest centimetre.

(2)

8. **(Question 8 - C2 June 2011)**

8.

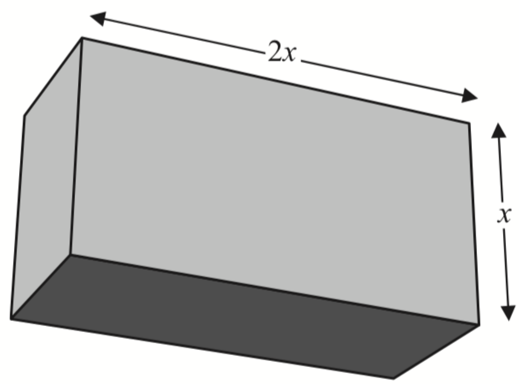

Figure 2

A cuboid has a rectangular cross-section where the length of the rectangle is equal to twice its width, x cm, as shown in Figure 2.
The volume of the cuboid is 81 cubic centimetres.

(a) Show that the total length, L cm, of the twelve edges of the cuboid is given by

$$L = 12x + \frac{162}{x^2}$$

(3)

(b) Use calculus to find the minimum value of L.

(6)

(c) Justify, by further differentiation, that the value of L that you have found is a minimum.

(2)

9. **(Question 10 - C2 January 2011)**

10. The volume V cm^3 of a box, of height x cm, is given by

$$V = 4x(5-x)^2, \quad 0 < x < 5$$

(a) Find $\frac{dV}{dx}$.
(4)

(b) Hence find the maximum volume of the box.
(4)

(c) Use calculus to justify that the volume that you found in part (b) is a maximum.
(2)

---

10. **(Question 3 - C2 June 2010)**

3. $y = x^2 - k\sqrt{x}$, where k is a constant.

(*a*) Find $\frac{dy}{dx}$.
(2)

(*b*) Given that y is decreasing at $x = 4$, find the set of possible values of k.
(2)

---

11. **(Question 9 - C2 January 2010)**

9. The curve C has equation $y = 12\sqrt{(x)} - x^{\frac{3}{2}} - 10, \quad x > 0$

(a) Use calculus to find the coordinates of the turning point on C.
(7)

(b) Find $\frac{d^2y}{dx^2}$.
(2)

(c) State the nature of the turning point.
(1)

Solutions

1. (a) $\frac{\pi x^2}{3}$
   (b) -
   (c) -
   (d) 120m
   (e) $\frac{d^2P}{dx^2} > 0$
2. (a) -
   (b) £483
   (c) $\frac{d^2C}{dr^2} > 0$
3. (a) -
   (b) -
   (c) 2880 cm^2
   (d) $\frac{d^2S}{dx^2} > 0$
4. (a) $(4, -28)$
   (b) Minimum
5. (a) -
   (b) $x = -\sqrt{2}$
   (c) $\frac{d^2y}{dx^2} = -48x^{-5}$
   (d) Maximum at P, Minimum at Q
6. (a) $h = \frac{60}{\pi x^2}$
   (b) -
   (c) $x = 2.12$
   (d) $A = 85$
   (e) -
7. (a) -
   (b) -
   (c) $P = 8$

(d) 21cm

8. (a) -
   (b) $L = 54$
   (c) $\frac{d^2S}{dx^2} > 0$

9. (a) $\frac{dV}{dx} = 100 - 80x + 12x^2$
   (b) $V = 74.1$
   (c) $\frac{d^2V}{dx^2} < 0$

10. (a) $\frac{dy}{dx} = 2x - \frac{1}{2}kx^{-\frac{1}{2}}$
    (b) $k > 32$

11. (a) $(4, 6)$
    (b) $\frac{d^2y}{dx^2} = -3x^{-\frac{3}{2}} - \frac{3}{4}x^{-\frac{1}{2}}$
    (c) Maximum