Inverse/Composite Functions - Past Exam Questions

1. **(Question 2 - C3 June 2018)**

2. The function f is defined by

$$f(x) = \frac{6}{2x+5} + \frac{2}{2x-5} + \frac{60}{4x^2-25}, \qquad x > 4$$

(a) Show that $f(x) = \dfrac{A}{Bx+C}$ where A, B and C are constants to be found. **(4)**

(b) Find $f^{-1}(x)$ and state its domain. **(3)**

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2. **(Question 3 - C3 June 2017)**

3.

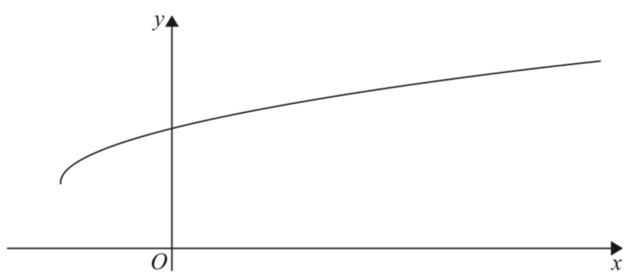

Figure 1

Figure 1 shows a sketch of part of the graph of $y = g(x)$, where

$$g(x) = 3 + \sqrt{x+2}, \qquad x \geqslant -2$$

(a) State the range of g. **(1)**

(b) Find $g^{-1}(x)$ and state its domain. **(3)**

(c) Find the exact value of x for which

$$g(x) = x$$

(4)

(d) Hence state the value of a for which

$$g(a) = g^{-1}(a)$$

(1)

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3. **(Question 7 - C3 June 2015)**

7.

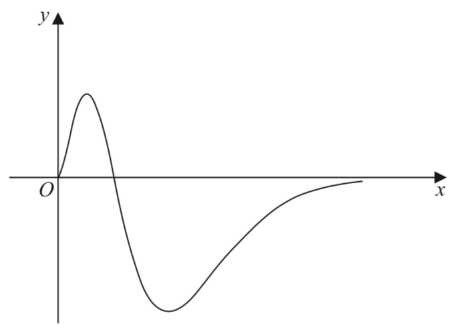

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$g(x) = x^2(1-x)e^{-2x}, \quad x \geqslant 0$$

(a) Show that $g'(x) = f(x)e^{-2x}$, where $f(x)$ is a cubic function to be found. **(3)**

(b) Hence find the range of g. **(6)**

(c) State a reason why the function $g^{-1}(x)$ does not exist. **(1)**

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4. **(Question 5 - C3 June 2014)**

5.

$$g(x) = \frac{x}{x+3} + \frac{3(2x+1)}{x^2+x-6}, \quad x > 3$$

(a) Show that $g(x) = \dfrac{x+1}{x-2}, \quad x > 3$ **(4)**

(b) Find the range of g. **(2)**

(c) Find the exact value of a for which $g(a) = g^{-1}(a)$. **(4)**

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5. **(Question 7 - C3 June 2013)**

7. The function f has domain $-2 \leqslant x \leqslant 6$ and is linear from $(-2, 10)$ to $(2, 0)$ and from $(2, 0)$ to $(6, 4)$. A sketch of the graph of $y = \mathrm{f}(x)$ is shown in Figure 1.

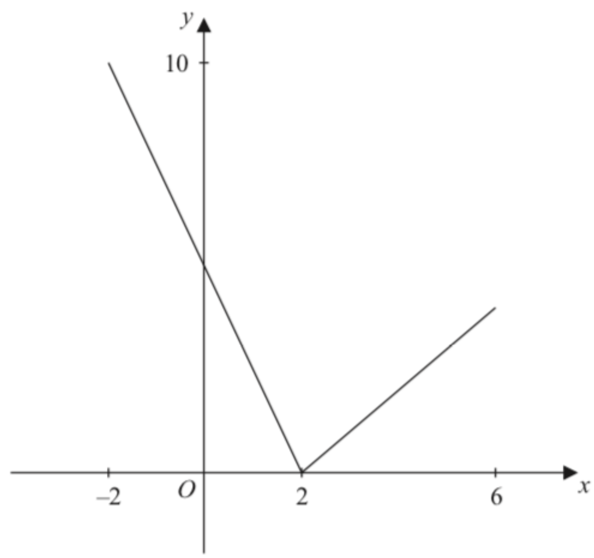

Figure 1

(a) Write down the range of f. **(1)**

(b) Find ff(0). **(2)**

The function g is defined by

$$\mathrm{g} : x \rightarrow \frac{4 + 3x}{5 - x}, \qquad x \in \mathbb{R}, \qquad x \neq 5$$

(c) Find $\mathrm{g}^{-1}(x)$ **(3)**

(d) Solve the equation $\mathrm{gf}(x) = 16$ **(5)**

6. **(Question 6 - C3 June 2012)**

6. The functions f and g are defined by

$$f : x \mapsto e^x + 2, \qquad x \in \mathbb{R}$$

$$g : x \mapsto \ln x, \qquad x > 0$$

(a) State the range of f.
(1)

(b) Find $fg(x)$, giving your answer in its simplest form.
(2)

(c) Find the exact value of x for which $f(2x+3) = 6$
(4)

(d) Find f^{-1}, the inverse function of f, stating its domain.
(3)

(e) On the same axes sketch the curves with equation $y = f(x)$ and $y = f^{-1}(x)$, giving the coordinates of all the points where the curves cross the axes.
(4)

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7. **(Question 4 - C3 June 2011)**

4. The function f is defined by

$$f : x \mapsto 4 - \ln(x+2), \quad x \in \mathbb{R}, \ x \geqslant -1$$

(a) Find $f^{-1}(x)$.
(3)

(b) Find the domain of f^{-1}.
(1)

The function g is defined by

$$g : x \mapsto e^{x^2} - 2, \quad x \in \mathbb{R}$$

(c) Find $fg(x)$, giving your answer in its simplest form.
(3)

(d) Find the range of fg.
(1)

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8. **(Question 4 - C3 June 2010)**

4. The function f is defined by

$$f : x \mapsto |2x - 5|, \quad x \in \mathbb{R}$$

(a) Sketch the graph with equation $y = f(x)$, showing the coordinates of the points where the graph cuts or meets the axes.

(2)

(b) Solve $f(x) = 15 + x$.

(3)

The function g is defined by

$$g : x \mapsto x^2 - 4x + 1, \quad x \in \mathbb{R}, \quad 0 \leqslant x \leqslant 5$$

(c) Find $fg(2)$.

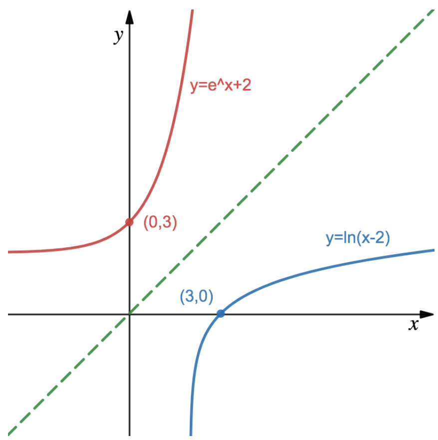
(2)

(d) Find the range of g.

(3)

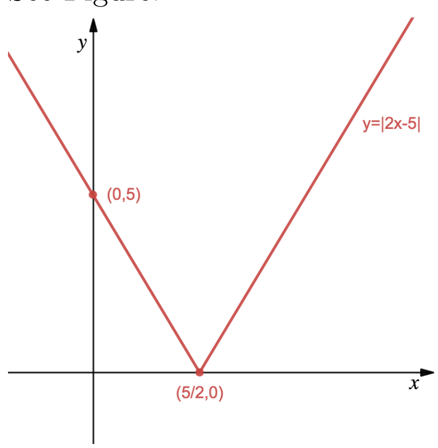
Solutions

1. (a) $A = 8$, $B = 2$, $C = -5$
   (b) $f^{-1}(x) = \frac{5x+8}{2x}$
2. (a) $y \geq 3$
   (b) $g^{-1}(x) = x^2 - 6x + 7 = (x-3)^2 - 2$, $x \geq 3$
   (c) $x = 1$, $x = 6$
   (d) $a = 6$
3. (a) $f(x) = 2x^3 - 5x^2 + 2x$
   (b) $-\frac{4}{e^4} \leq g(x) \leq \frac{1}{8e}$
   (c) $g(x)$ is not a one-to-one function
4. (a) -
   (b) $1 < y < 4$
   (c) $x = \frac{3+\sqrt{13}}{2}$
5. (a) $0 \leq f(x) \leq 10$
   (b) $ff(0) = 3$
   (c) $g^{-1}(x) = \frac{5x-4}{3+x}$
   (d) $x = 6$, $x = 0.4$
6. (a) $f(x) > 2$
   (b) $fg(x) = x + 2$
   (c) $x = \frac{\ln 4 - 3}{2}$
   (d) $f^{-1}(x) = \ln(x-2)$, $x > 2$
   (e) See Figure:



7. (a) $f^{-1}(x) = e^{4-x} - 2$
   (b) $x \leq 4$
   (c) $fg(x) = 4 - x^2$
   (d) $fg(x) \leq 4$

8. (a) See Figure:



   (b) $x = 20,\ x = -\frac{10}{3}$
   (c) $fg(2) = 11$
   (d) $-3 \leq g(x) \leq 6$