Numerical Methods - Past Edexcel Exam Questions

1. **(Question 4 - C3 June 2018)**

4.

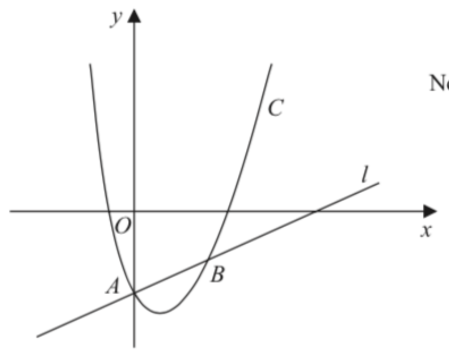

Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = e^{-2x} + x^2 - 3$$

The curve C crosses the y-axis at the point A.

The line l is the normal to C at the point A.

(a) Find the equation of l, writing your answer in the form $y = mx + c$, where m and c are constants.

(5)

The line l meets C again at the point B, as shown in Figure 1.

(b) Show that the x coordinate of B is a solution of

$$x = \sqrt{1 + \frac{1}{2}x - e^{-2x}}$$

(2)

Using the iterative formula

$$x_{n+1} = \sqrt{1 + \frac{1}{2}x_n - e^{-2x_n}}$$

with $x_1 = 1$

(c) find x_2 and x_3 to 3 decimal places.

(2)

2. **(Question 5 - C3 June 2017)**

5.

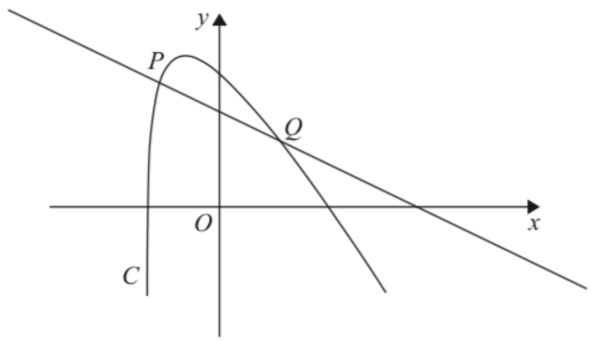

Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = 2\ln(2x + 5) - \frac{3x}{2}, \qquad x > -2.5$$

The point P with x coordinate -2 lies on C.

(a) Find an equation of the normal to C at P. Write your answer in the form $ax + by = c$, where a, b and c are integers.

(5)

The normal to C at P cuts the curve again at the point Q, as shown in Figure 2.

(b) Show that the x coordinate of Q is a solution of the equation

$$x = \frac{20}{11}\ln(2x + 5) - 2$$

(3)

The iteration formula

$$x_{n+1} = \frac{20}{11}\ln(2x_n + 5) - 2$$

can be used to find an approximation for the x coordinate of Q.

(c) Taking $x_1 = 2$, find the values of x_2 and x_3, giving each answer to 4 decimal places.

(2)

3. **(Question 4 - C3 June 2016)**

4.

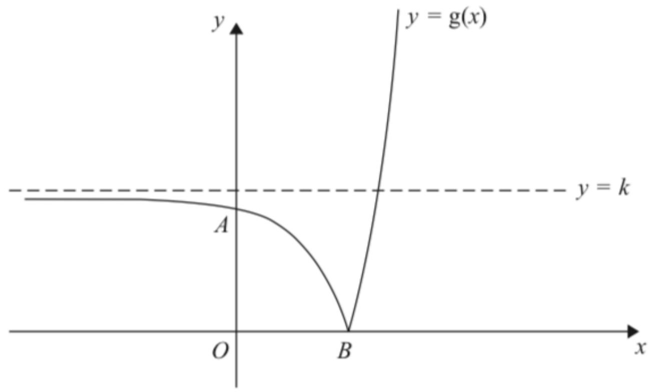

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = |4\mathrm{e}^{2x} - 25|, \quad x \in \mathbb{R}$$

The curve cuts the y-axis at the point A and meets the x-axis at the point B. The curve has an asymptote $y = k$, where k is a constant, as shown in Figure 1

(a) Find, giving each answer in its simplest form,

(i) the y coordinate of the point A,

(ii) the exact x coordinate of the point B,

(iii) the value of the constant k.

(5)

The equation $\mathrm{g}(x) = 2x + 43$ has a positive root at $x = \alpha$

(b) Show that α is a solution of $x = \frac{1}{2}\ln\left(\frac{1}{2}x + 17\right)$

(2)

The iteration formula

$$x_{n+1} = \frac{1}{2}\ln\left(\frac{1}{2}x_n + 17\right)$$

can be used to find an approximation for α

(c) Taking $x_0 = 1.4$ find the values of x_1 and x_2
Give each answer to 4 decimal places.

(2)

(d) By choosing a suitable interval, show that $\alpha = 1.437$ to 3 decimal places.

(2)

4. **(Question 6 - C3 June 2015)**

6.

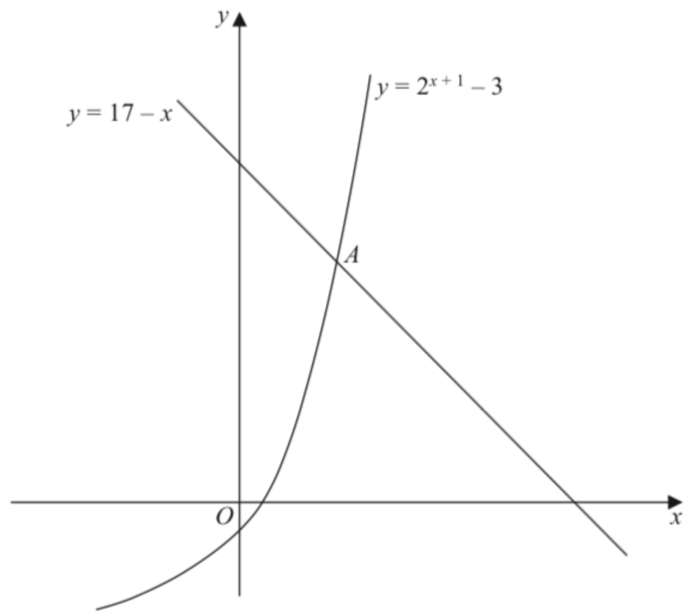

Figure 1

Figure 1 is a sketch showing part of the curve with equation $y = 2^{x+1} - 3$ and part of the line with equation $y = 17 - x$.

The curve and the line intersect at the point A.

(a) Show that the x coordinate of A satisfies the equation

$$x = \frac{\ln(20 - x)}{\ln 2} - 1$$

(3)

(b) Use the iterative formula

$$x_{n+1} = \frac{\ln(20 - x_n)}{\ln 2} - 1, \quad x_0 = 3$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

(c) Use your answer to part (b) to deduce the coordinates of the point A, giving your answers to one decimal place.

(2)

5. **(Question 6 - C3 June 2014)**

6.

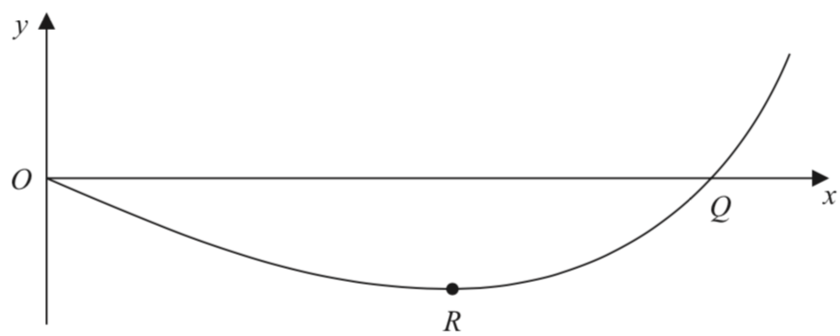

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = 2\cos\left(\frac{1}{2}x^2\right) + x^3 - 3x - 2$$

The curve crosses the x-axis at the point Q and has a minimum turning point at R.

(a) Show that the x coordinate of Q lies between 2.1 and 2.2

(2)

(b) Show that the x coordinate of R is a solution of the equation

$$x = \sqrt{1 + \frac{2}{3}x\sin\left(\frac{1}{2}x^2\right)}$$

(4)

Using the iterative formula

$$x_{n+1} = \sqrt{1 + \frac{2}{3}x_n\sin\left(\frac{1}{2}x_n^{\ 2}\right)}, \quad x_0 = 1.3$$

(c) find the values of x_1 and x_2 to 3 decimal places.

(2)

6. **(Question 4 - C3 June 2013)**

4. $$f(x) = 25x^2e^{2x} - 16, \qquad x \in \mathbb{R}$$

(a) Using calculus, find the exact coordinates of the turning points on the curve with equation $y = f(x)$.

(5)

(b) Show that the equation $f(x) = 0$ can be written as $x = \pm\frac{4}{5}e^{-x}$

(1)

The equation $f(x) = 0$ has a root α, where $\alpha = 0.5$ to 1 decimal place.

(c) Starting with $x_0 = 0.5$, use the iteration formula

$$x_{n+1} = \frac{4}{5}e^{-x_n}$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

(d) Give an accurate estimate for α to 2 decimal places, and justify your answer.

(2)

---

7. **(Question 2 - C3 June 2012)**

2. $$f(x) = x^3 + 3x^2 + 4x - 12$$

(a) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{4(3-x)}{(3+x)}\right)}, \qquad x \neq -3$$

(3)

The equation $x^3 + 3x^2 + 4x - 12 = 0$ has a single root which is between 1 and 2

(b) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{4(3-x_n)}{(3+x_n)}\right)}, \; n \geqslant 0$$

with $x_0 = 1$ to find, to 2 decimal places, the value of x_1, x_2 and x_3.

(3)

The root of $f(x) = 0$ is α.

(c) By choosing a suitable interval, prove that $\alpha = 1.272$ to 3 decimal places.

(3)

---

8. **(Question 2 - C3 June 2011)**

2. $$f(x) = 2\sin(x^2) + x - 2, \qquad 0 \leqslant x < 2\pi$$

(a) Show that $f(x) = 0$ has a root α between $x = 0.75$ and $x = 0.85$ **(2)**

The equation $f(x) = 0$ can be written as $x = \left[\arcsin(1 - 0.5x)\right]^{\frac{1}{2}}$.

(b) Use the iterative formula

$$x_{n+1} = \left[\arcsin(1 - 0.5x_n)\right]^{\frac{1}{2}}, \quad x_0 = 0.8$$

to find the values of x_1, x_2 and x_3, giving your answers to 5 decimal places. **(3)**

(c) Show that $\alpha = 0.80157$ is correct to 5 decimal places. **(3)**

---

9. **(Question 3 - C3 June 2010)**

3. $$f(x) = 4\operatorname{cosec} x - 4x + 1, \quad \text{where } x \text{ is in radians.}$$

(a) Show that there is a root α of $f(x) = 0$ in the interval $[1.2, 1.3]$. **(2)**

(b) Show that the equation $f(x) = 0$ can be written in the form

$$x = \frac{1}{\sin x} + \frac{1}{4}$$ **(2)**

(c) Use the iterative formula

$$x_{n+1} = \frac{1}{\sin x_n} + \frac{1}{4}, \quad x_0 = 1.25,$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 4 decimal places. **(3)**

(d) By considering the change of sign of $f(x)$ in a suitable interval, verify that $\alpha = 1.291$ correct to 3 decimal places. **(2)**

Solutions

1. (a) $y = \frac{1}{2}x - 3$
   (b) -
   (c) $x_2 = 1.168$, $x_3 = 1.220$
2. (a) $2x + 5y = 11$
   (b) -
   (c) $x_2 = 1.9950$, $x_3 = 1.9929$
3. (a) i. $y = 21$
   ii. $x = \ln \frac{5}{2}$
   iii. $k = 25$
   (b) -
   (c) $x_1 = 1.4368$, $x_2 = 1.4373$
   (d) -
4. (a) -
   (b) $x_1 = 3.087$, $x_2 = 3.080$, $x_3 = 3.081$
   (c) $(3.1, 13.9)$
5. (a) At $x = 2.1$, $y = -0.22 < 0$ and at $x = 2.2$, $y = 0.55 > 0$. There must be a root in between 2.1 and 2.2.
   (b) -
   (c) $x_1 = 1.284$, $x_2 = 1.276$
6. (a) $(0, -16)$, $(-1, 25e^{-2} - 16)$
   (b) -
   (c) $x_1 = 0.485$, $x_2 = 0.492$, $x_3 = 0.489$
   (d) $f(0.485) < 0$ and $f(0.495) > 0$ so $\alpha = 0.49$ to 2 d.p.
7. (a) -
   (b) $x_1 = 1.41$, $x_2 = 1.20$, $x_3 = 1.31$

(c) $f(1.2715) < 0$ and $f(1.2725) > 0$ so $\alpha = 1.272$ to 3 d.p.

8. (a) $f(0.75) < 0$ and $f(0.85) > 0$ so there must be a root in between.
   (b) $x_1 = 0.80219$, $x_2 = 0.80133$, $x_3 = 0.80167$
   (c) $f(0.801565) < 0$ and $f(0.801575) > 0$ so $\alpha = 0.80157$ to 5 d.p.

9. (a) $f(1.2) > 0$ and $f(1.3) < 0$ so there must be a root in between.
   (b) -
   (c) $x_1 = 1.3038$, $x_2 = 1.2867$, $x_3 = 1.2917$
   (d) $f(1.2905) > 0$ and $f(1.2915) < 0$ so $\alpha = 1.291$ to 3 d.p.