

## Integration by Parts - Past Edexcel Exam Questions

1.

(Question 8 - C4 June 2018)

8.

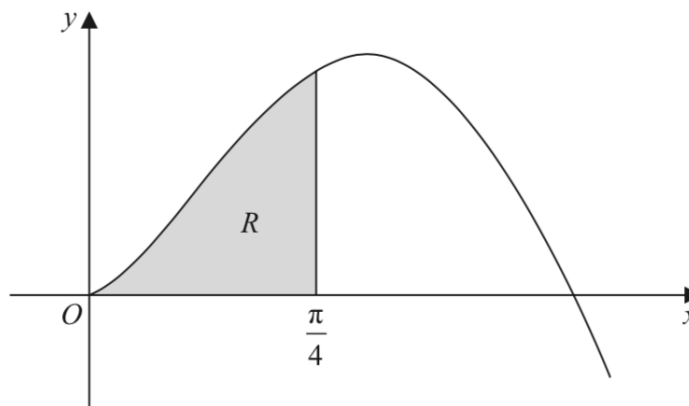


Diagram not drawn to scale

**Figure 3**

(a) Find  $\int x \cos 4x \, dx$  (3)

Figure 3 shows part of the curve with equation  $y = \sqrt{x} \sin 2x$ ,  $x \geq 0$

The finite region  $R$ , shown shaded in Figure 3, is bounded by the curve, the  $x$ -axis and the line with equation  $x = \frac{\pi}{4}$

The region  $R$  is rotated through  $2\pi$  radians about the  $x$ -axis to form a solid of revolution.

(b) Find the exact value of the volume of this solid of revolution, giving your answer in its simplest form.  
*(Solutions based entirely on graphical or numerical methods are not acceptable.)* (6)

2.

(Question 2 - C4 June 2016)

2.

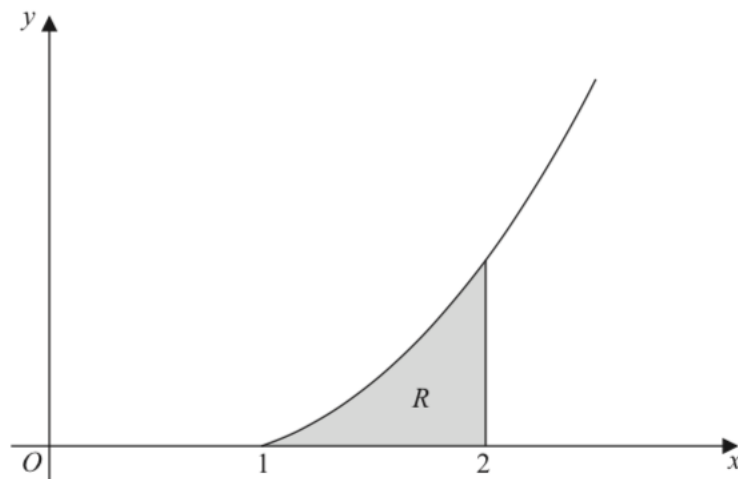


Figure 1

Figure 1 shows a sketch of part of the curve with equation  $y = x^2 \ln x$ ,  $x \geq 1$

The finite region  $R$ , shown shaded in Figure 1, is bounded by the curve, the  $x$ -axis and the line  $x = 2$

The table below shows corresponding values of  $x$  and  $y$  for  $y = x^2 \ln x$

$x$	1	1.2	1.4	1.6	1.8	2
$y$	0	0.2625		1.2032	1.9044	2.7726

- (a) Complete the table above, giving the missing value of  $y$  to 4 decimal places. (1)
- (b) Use the trapezium rule with all the values of  $y$  in the completed table to obtain an estimate for the area of  $R$ , giving your answer to 3 decimal places. (3)
- (c) Use integration to find the exact value for the area of  $R$ . (5)

3.

(Question 3 - C4 June 2015)

3.

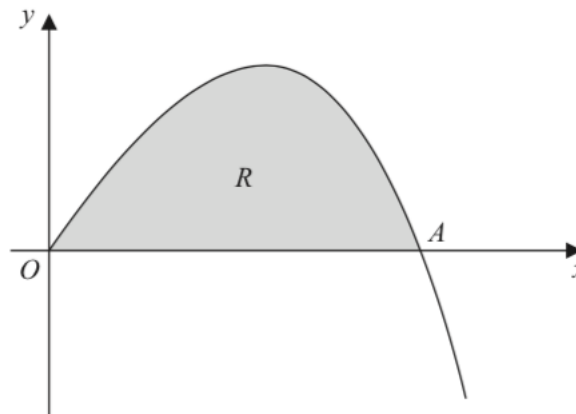


Figure 1

Figure 1 shows a sketch of part of the curve with equation  $y = 4x - xe^{\frac{1}{2}x}$ ,  $x \geq 0$

The curve meets the  $x$ -axis at the origin  $O$  and cuts the  $x$ -axis at the point  $A$ .

(a) Find, in terms of  $\ln 2$ , the  $x$  coordinate of the point  $A$ . (2)

(b) Find

$$\int xe^{\frac{1}{2}x} dx$$
(3)

The finite region  $R$ , shown shaded in Figure 1, is bounded by the  $x$ -axis and the curve with equation

$$y = 4x - xe^{\frac{1}{2}x}, \quad x \geq 0$$

(c) Find, by integration, the exact value for the area of  $R$ .  
Give your answer in terms of  $\ln 2$  (3)

4.

(Question 6 - C4 June 2014)

6. (i) Find

$$\int x e^{4x} dx \quad (3)$$

(ii) Find

$$\int \frac{8}{(2x-1)^3} dx, \quad x > \frac{1}{2} \quad (2)$$

(iii) Given that  $y = \frac{\pi}{6}$  at  $x = 0$ , solve the differential equation

$$\frac{dy}{dx} = e^x \operatorname{cosec} 2y \operatorname{cosec} y \quad (7)$$

5.

(Question 1 - C4 June 2013)

1. (a) Find  $\int x^2 e^x dx$ . (5)

(b) Hence find the exact value of  $\int_0^1 x^2 e^x dx$ . (2)

## Solutions

1. (a)  $\frac{1}{4}x \sin(4x) + \frac{1}{16} \cos(4x) + c$   
(b) NOT EXAMINABLE
2. (a) 0.6595  
(b) 1.083  
(c)  $\frac{8}{3} \ln(2) - \frac{7}{9}$
3. (a)  $4 \ln(2)$   
(b)  $2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x} + c$   
(c)  $32(\ln 2)^2 - 32 \ln(2) + 12$
4. (a)  $\frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x} + c$   
(b)  $\frac{-2}{(2x-1)^2} + c$   
(c)  $\frac{1}{2} \sin(y) - \frac{1}{6} \sin(3y) = e^x - \frac{11}{12}$
5. (a)  $(x^2 - 2x + 2)e^x + c$   
(b)  $e - 2$