

A2 Partial Fractions - Past Edexcel Exam Questions

1. (Question 3 - C4 June 2018)

3. (i) Given that

$$\frac{13 - 4x}{(2x + 1)^2(x + 3)} \equiv \frac{A}{(2x + 1)} + \frac{B}{(2x + 1)^2} + \frac{C}{(x + 3)}$$

(a) find the values of the constants A , B and C . (4)

(b) Hence find

$$\int \frac{13 - 4x}{(2x + 1)^2(x + 3)} dx, \quad x > -\frac{1}{2}$$

(3)

(ii) Find

$$\int (e^x + 1)^3 dx$$

(3)

(iii) Using the substitution $u^3 = x$, or otherwise, find

$$\int \frac{1}{4x + 5x^{\frac{1}{3}}} dx, \quad x > 0$$

(4)

2. (Question 1 - C3 June 2017)

1. Express $\frac{4x}{x^2 - 9} - \frac{2}{x + 3}$ as a single fraction in its simplest form. (4)

3. (Question 6 - C3 June 2016)

6.
$$f(x) = \frac{x^4 + x^3 - 3x^2 + 7x - 6}{x^2 + x - 6}, \quad x > 2, x \in \mathbb{R}$$

(a) Given that

$$\frac{x^4 + x^3 - 3x^2 + 7x - 6}{x^2 + x - 6} \equiv x^2 + A + \frac{B}{x - 2}$$

find the values of the constants A and B .

(4)

(b) Hence or otherwise, using calculus, find an equation of the normal to the curve with equation $y = f(x)$ at the point where $x = 3$

(5)

4. (Question 6 - C4 June 2016)

6. (i) Given that $y > 0$, find

$$\int \frac{3y - 4}{y(3y + 2)} dy$$

(6)

(ii) (a) Use the substitution $x = 4 \sin^2 \theta$ to show that

$$\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)} dx = \lambda \int_0^{\frac{\pi}{3}} \sin^2 \theta d\theta$$

where λ is a constant to be determined.

(5)

(b) Hence use integration to find

$$\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)} dx$$

giving your answer in the form $a\pi + b$, where a and b are exact constants.

(4)

5. (Question 7 - C4 June 2015)

7. (a) Express $\frac{2}{P(P-2)}$ in partial fractions. (3)

A team of biologists is studying a population of a particular species of animal.

The population is modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{2}P(P-2)\cos 2t, \quad t \geq 0$$

where P is the population in thousands, and t is the time measured in years since the start of the study.

Given that $P = 3$ when $t = 0$,

(b) solve this differential equation to show that

$$P = \frac{6}{3 - e^{\frac{1}{2}\sin 2t}} \quad (7)$$

(c) find the time taken for the population to reach 4000 for the first time.
Give your answer in years to 3 significant figures. (3)

6. (Question 1 - C3 June 2013)

1. Given that

$$\frac{3x^4 - 2x^3 - 5x^2 - 4}{x^2 - 4} \equiv ax^2 + bx + c + \frac{dx + e}{x^2 - 4}, \quad x \neq \pm 2$$

find the values of the constants a , b , c , d and e . (4)

7. (Question 1 - C3 June 2012)

1. Express

$$\frac{2(3x+2)}{9x^2-4} - \frac{2}{3x+1}$$

as a single fraction in its simplest form.

(4)

8. (Question 1 - C4 June 2012)

1.
$$f(x) = \frac{1}{x(3x-1)^2} = \frac{A}{x} + \frac{B}{(3x-1)} + \frac{C}{(3x-1)^2}$$

(a) Find the values of the constants A , B and C .

(4)

(b) (i) Hence find $\int f(x) dx$.

(ii) Find $\int_1^2 f(x) dx$, leaving your answer in the form $a + \ln b$,
where a and b are constants.

(6)

9. (Question 7 - C3 June 2011)

7.
$$f(x) = \frac{4x-5}{(2x+1)(x-3)} - \frac{2x}{x^2-9}, \quad x \neq \pm 3, x \neq -\frac{1}{2}$$

(a) Show that

$$f(x) = \frac{5}{(2x+1)(x+3)}$$

(5)

The curve C has equation $y=f(x)$. The point $P\left(-1, -\frac{5}{2}\right)$ lies on C .

(b) Find an equation of the normal to C at P .

(8)

10. (Question 1 - C4 June 2011)

1.
$$\frac{9x^2}{(x-1)^2(2x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{2x+1}$$

Find the values of the constants A , B and C .

(4)

11. (Question 5 - C4 June 2010)

5.
$$\frac{2x^2 + 5x - 10}{(x-1)(x+2)} \equiv A + \frac{B}{x-1} + \frac{C}{x+2}$$

(a) Find the values of the constants A , B and C .

(4)

(b) Hence, or otherwise, expand $\frac{2x^2 + 5x - 10}{(x-1)(x+2)}$ in ascending powers of x , as far as the term in x^2 . Give each coefficient as a simplified fraction.

(7)

Solutions

1. (a) i. $A = -2, B = 6, C = 1$
 ii. $-\ln(2x + 1) - \frac{3}{2x+1} + \ln(x + 3) + c$
- (b) i. $\frac{1}{3}e^{3x} + \frac{3}{2}e^{2x} + 3e^x + x + c$
 ii. $\frac{3}{8} \ln(4x^{\frac{2}{3}} + 5) + c$
2. $\frac{2}{x-3}$
3. (a) $A = 3, B = 4$
 (b) $y = -\frac{1}{2}x + \frac{35}{2}$
4. (a) $-2 \ln y + 3 \ln(3y + 2) + c$
 (b) i. $\lambda = 8$
 ii. $\frac{4\pi}{3} - \sqrt{3}$
5. (a) $\frac{1}{p-2} - \frac{1}{p}$
 (b) -
 (c) $t = 0.473$
6. $a = 3, b = -2, c = 7, d = -8, e = 24$
7. $\frac{6}{(3x-4)(3x+1)}$
8. (a) $A = 1, B = -3, C = 3$
 (b) i. $\ln|x| - \ln|3x - 1| - \frac{1}{3x-1} + c$
 ii. $\frac{3}{10} + \ln \frac{4}{5}$
9. (a) -
 (b) $y = \frac{4}{15}x - \frac{67}{30}$
10. $A = 4, B = 3, C = 1$
11. (a) $A = 2, B = -1, C = 4$
 (b) $5 + \frac{3}{2}x^2$