A2 Trigonometric Integration - Past Edexcel Exam Questions

1. **(Question 6 - C4 June 2018)**

6. Given that $y = 2$ when $x = -\frac{\pi}{8}$, solve the differential equation

$$\frac{dy}{dx} = \frac{y^2}{3\cos^2 2x} \qquad -\frac{1}{2} < x < \frac{1}{2}$$

giving your answer in the form $y = f(x)$.

(6)

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2. **(Question 6 - C4 June 2014)**

6. (i) Find

$$\int x e^{4x} dx$$

(3)

(ii) Find

$$\int \frac{8}{(2x-1)^3} dx, \quad x > \frac{1}{2}$$

(2)

(iii) Given that $y = \frac{\pi}{6}$ at $x = 0$, solve the differential equation

$$\frac{dy}{dx} = e^x \operatorname{cosec} 2y \operatorname{cosec} y$$

(7)

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3. **(Question 8 - C4 June 2017)**

8.

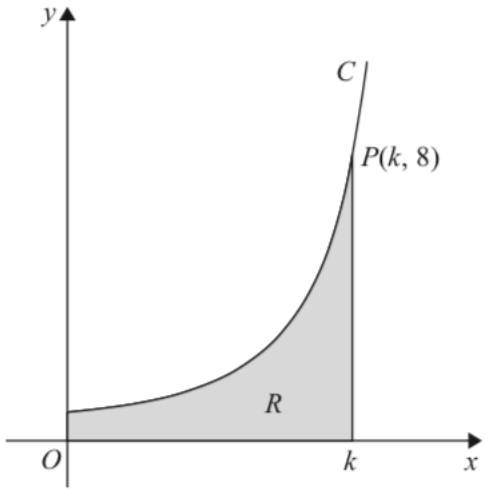

Diagram not drawn to scale

Figure 4

Figure 4 shows a sketch of part of the curve C with parametric equations

$$x = 3\theta\sin\theta, \qquad y = \sec^3\theta, \qquad 0 \leqslant \theta < \frac{\pi}{2}$$

The point $P(k, 8)$ lies on C, where k is a constant.

(a) Find the exact value of k.

(2)

The finite region R, shown shaded in Figure 4, is bounded by the curve C, the y-axis, the x-axis and the line with equation $x = k$.

(b) Show that the area of R can be expressed in the form

$$\lambda\int_{\alpha}^{\beta}\left(\theta\sec^2\theta + \tan\theta\sec^2\theta\right)\mathrm{d}\theta$$

where λ, α and β are constants to be determined.

(4)

(c) Hence use integration to find the exact value of the area of R.

(6)

4. **(Question 3 - C4 June 2013)**

3.

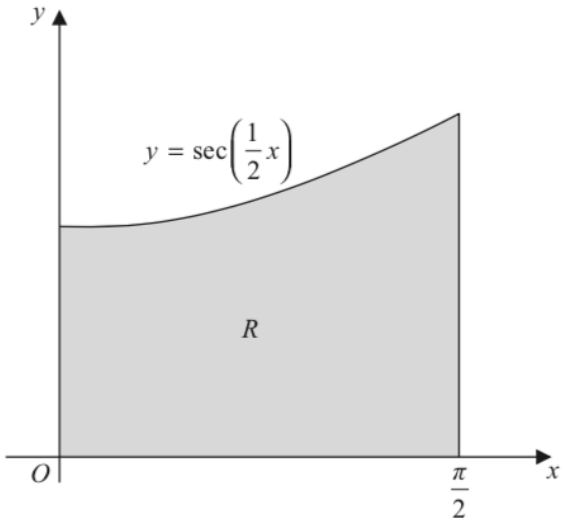

Figure 1

Figure 1 shows the finite region R bounded by the x-axis, the y-axis, the line $x = \frac{\pi}{2}$ and the curve with equation

$$y = \sec\left(\frac{1}{2}x\right), \quad 0 \leqslant x \leqslant \frac{\pi}{2}$$

The table shows corresponding values of x and y for $y = \sec\left(\frac{1}{2}x\right)$.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
y	1	1.035276		1.414214

(a) Complete the table above giving the missing value of y to 6 decimal places.

(1)

(b) Using the trapezium rule, with all of the values of y from the completed table, find an approximation for the area of R, giving your answer to 4 decimal places.

(3)

Region R is rotated through 2π radians about the x-axis.

(c) Use calculus to find the exact volume of the solid formed.

(4)

5. **(Question 4 - C4 June 2012)**

4. Given that $y = 2$ at $x = \frac{\pi}{4}$, solve the differential equation

$$\frac{dy}{dx} = \frac{3}{y\cos^2 x}$$

(5)

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6. **(Question 6 - C4 June 2010)**

6.

$$f(\theta) = 4\cos^2\theta - 3\sin^2\theta$$

(a) Show that $f(\theta) = \frac{1}{2} + \frac{7}{2}\cos 2\theta$.

(3)

(b) Hence, using calculus, find the exact value of $\int_0^{\frac{\pi}{2}} \theta f(\theta)\, d\theta$.

(7)

Solutions

1. $y = \frac{6}{2-\tan(2x)}$
2. (a) $\frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x} + c$
   (b) $\frac{-2}{(2x-1)^2} + c$
   (c) $\frac{1}{2}\sin(y) - \frac{1}{6}\sin(3y) = e^x - \frac{11}{12}$
3. (a) $k = \frac{\sqrt{3}\pi}{2}$
   (b) $\lambda = 3$, $\alpha = 0$, $\beta = \frac{\pi}{3}$
   (c) $\sqrt{3}\pi - 3\ln(2) + \frac{9}{2}$
4. (a) 1.154701
   (b) 1.7787
   (c) NOT EXAMINABLE
5. $\frac{y^2}{2} = 3\tan(x) - 1$ or $y = \sqrt{6\tan(x) - 2}$
6. (a) -
   (b) $\frac{\pi^2}{16} - \frac{7}{4}$