Exponentials - Past Edexcel Exam Questions

1. **(Question 3 - C3 June 2018)**

3. The value of a car is modelled by the formula

$$V = 16\,000e^{-kt} + A, \qquad t \geqslant 0, t \in \mathbb{R}$$

where V is the value of the car in pounds, t is the age of the car in years, and k and A are positive constants.

Given that the value of the car is £17 500 when new and £13 500 two years later,

(a) find the value of A,

(1)

(b) show that $k = \ln\left(\frac{2}{\sqrt{3}}\right)$

(4)

(c) Find the age of the car, in years, when the value of the car is £6000

Give your answer to 2 decimal places.

(4)

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2. **(Question 2 - C3 June 2017)**

2. Find the exact solutions, in their simplest form, to the equations

(a) $e^{3x-9} = 8$

(3)

(b) $\ln(2y + 5) = 2 + \ln(4 - y)$

(4)

3. **(Question 9 - C3 June 2016)**

9. The amount of an antibiotic in the bloodstream, from a given dose, is modelled by the formula

$$x = De^{-0.2t}$$

where x is the amount of the antibiotic in the bloodstream in milligrams, D is the dose given in milligrams and t is the time in hours after the antibiotic has been given.

A first dose of 15 mg of the antibiotic is given.

(a) Use the model to find the amount of the antibiotic in the bloodstream 4 hours after the dose is given. Give your answer in mg to 3 decimal places.

(2)

A second dose of 15 mg is given 5 hours after the first dose has been given. Using the same model for the second dose,

(b) show that the **total** amount of the antibiotic in the bloodstream 2 hours after the second dose is given is 13.754 mg to 3 decimal places.

(2)

No more doses of the antibiotic are given. At time T hours after the second dose is given, the total amount of the antibiotic in the bloodstream is 7.5 mg.

(c) Show that $T = a\ln\left(b + \frac{b}{e}\right)$, where a and b are integers to be determined.

(4)

4. **(Question 4 - C3 June 2015)**

4. Water is being heated in an electric kettle. The temperature, θ °C, of the water t seconds after the kettle is switched on, is modelled by the equation

$$\theta = 120 - 100e^{-\lambda t}, \qquad 0 \leqslant t \leqslant T$$

(a) State the value of θ when $t = 0$

(1)

Given that the temperature of the water in the kettle is 70 °C when $t = 40$,

(b) find the exact value of λ, giving your answer in the form $\frac{\ln a}{b}$, where a and b are integers.

(4)

When $t = T$, the temperature of the water reaches 100 °C and the kettle switches off.

(c) Calculate the value of T to the nearest whole number.

(2)

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5. **(Question 2 - C3 June 2014)**

2. Find the exact solutions, in their simplest form, to the equations

(a) $2\ln(2x + 1) - 10 = 0$

(2)

(b) $3^x e^{4x} = e^7$

(4)

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6. **(Question 6 - C3 June 2013)**

6. Find algebraically the exact solutions to the equations

(a) $\ln(4 - 2x) + \ln(9 - 3x) = 2\ln(x + 1), \quad -1 < x < 2$ **(5)**

(b) $2^x e^{3x+1} = 10$

Give your answer to (b) in the form $\frac{a + \ln b}{c + \ln d}$ where a, b, c and d are integers. **(5)**

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7. **(Question 8 - C3 January 2013)**

8. The value of Bob's car can be calculated from the formula

$$V = 17000e^{-0.25t} + 2000e^{-0.5t} + 500$$

where V is the value of the car in pounds (£) and t is the age in years.

(a) Find the value of the car when $t = 0$ **(1)**

(b) Calculate the exact value of t when $V = 9500$ **(4)**

(c) Find the rate at which the value of the car is decreasing at the instant when $t = 8$. Give your answer in pounds per year to the nearest pound. **(4)**

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8. **(Question 3 - C3 January 2012)**

3. The area, A mm^2, of a bacterial culture growing in milk, t hours after midday, is given by

$$A = 20\,e^{1.5t}, \quad t \geqslant 0$$

(a) Write down the area of the culture at midday.

(1)

(b) Find the time at which the area of the culture is twice its area at midday. Give your answer to the nearest minute.

(5)

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9. **(Question 5 - C3 June 2011)**

5. The mass, m grams, of a leaf t days after it has been picked from a tree is given by

$$m = p\,e^{-kt}$$

where k and p are positive constants.

When the leaf is picked from the tree, its mass is 7.5 grams and 4 days later its mass is 2.5 grams.

(a) Write down the value of p.

(1)

(b) Show that $k = \frac{1}{4}\ln 3$.

(4)

(c) Find the value of t when $\frac{dm}{dt} = -0.6\ln 3$.

(6)

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10. **(Question 4 - C3 January 2011)**

4. Joan brings a cup of hot tea into a room and places the cup on a table. At time t minutes after Joan places the cup on the table, the temperature, θ°C, of the tea is modelled by the equation

$$\theta = 20 + Ae^{-kt},$$

where A and k are positive constants.

Given that the initial temperature of the tea was 90°C,

(a) find the value of A.

(2)

The tea takes 5 minutes to decrease in temperature from 90°C to 55°C.

(b) Show that $k = \frac{1}{5}\ln 2$.

(3)

(c) Find the rate at which the temperature of the tea is decreasing at the instant when $t = 10$. Give your answer, in °C per minute, to 3 decimal places.

(3)

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11. **(Question 8 - C3 June 2010)**

8. (a) Simplify fully

$$\frac{2x^2 + 9x - 5}{x^2 + 2x - 15}$$

(3)

Given that

$$\ln(2x^2 + 9x - 5) = 1 + \ln(x^2 + 2x - 15), \quad x \neq -5,$$

(b) find x in terms of e.

(4)

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12. **(Question 9 - C3 January 2010)**

9. (i) Find the exact solutions to the equations

(*a*) $\ln(3x - 7) = 5$,

(3)

(*b*) $3^x e^{7x+2} = 15$.

(5)

Part ii) of this question is a functions question.

Solutions

1. (a) $A = 1500$
   (b) -
   (c) 8.82 years
2. (a) $x = \frac{\ln(8)+9}{3}$ or $x = \ln(2) + 3$
   (b) $y = \frac{4e^2-5}{e^2+2}$
3. (a) 6.740mg
   (b) -
   (c) $a = 5$, $b = 2$
4. (a) $\theta = 20$
   (b) $\lambda = \frac{\ln(2)}{40}$
   (c) $T = 93$
5. (a) $x = \frac{e^5-1}{2}$
   (b) $x = \frac{7}{4+\ln(3)}$
6. (a) $x = \frac{7}{5}$
   (b) $x = \frac{-1+\ln(10)}{3+\ln(2)}$
7. (a) £$19,500$
   (b) $4\ln(2)$
   (c) £593 per year
8. (a) $A = 20\text{ mm}^2$
   (b) 12:28pm
9. (a) $p = 7.5$
   (b) -
   (c) $t = 4.15$ to 2 d.p.
10. (a) $A = 70$

(b) -

(c) 2.426

11. (a) $\frac{2x-1}{x-3}$

(b) $x = \frac{1-3e}{2-e}$

12. (a) $x = \frac{e^5+7}{3}$

(b) $x = \frac{\ln(15)-2}{\ln(3)+7}$